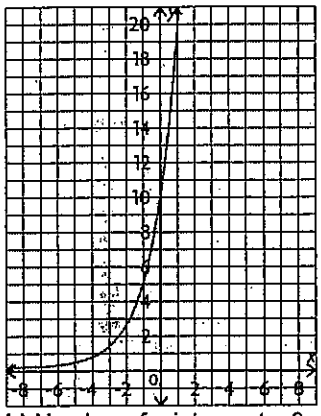
Chapter 6: Exponential Functions

Lesson 6.1: Exploring the Characteristics of Exponential Functions, page 337

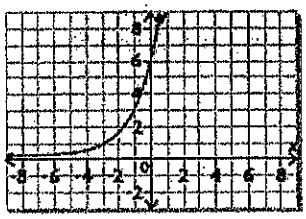
1. a) No, linear
b) Yes
c) No, quadratic
d) No, cubic
e) Yes
f) No, quadratic

2. b) No x-intercepts; y-intercept: $y = 1$
End behaviour: QII to QI
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
e) No x-intercepts; y-intercept: $y = 1$
End behaviour: QII to QI
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$

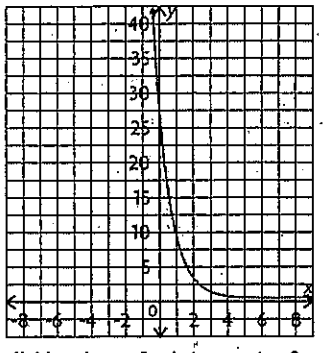
3. a) Number of x-intercepts: 0;
y-intercept: $y = 10$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



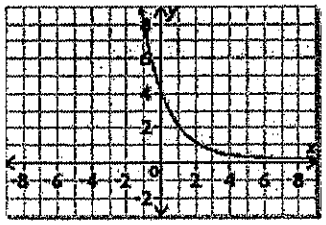
b) Number of x-intercepts: 0
y-intercept: $y = 6$
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



c) Number of x-intercepts: 0
y-intercept: $y = 27$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



d) Number of x-intercepts: 0
y-intercept: $y = 4$
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



Lesson 6.2: Relating the Characteristics of an Exponential Function to Its Equation, page 346

1. a)

Term	First Difference	Second Difference
14 – 7	7	
28 – 14	14	7
56 – 28	28	14
112 – 56	56	28
224 – 112	112	56

I noticed that the difference between consecutive y-values doubles from one pair of values to the next. I know that there must be a base value of 2 being raised to an exponent and then multiplied by some constant term in the equation. The equation looks like $y = a(2^x)$. Thus, the answer is yes, because for each unit increase in x, the value of y doubles.

b)

Term	First Difference	Second Difference
3072 – 768	2304	
768 – 192	576	1728
192 – 48	144	432
48 – 12	36	108
12 – 3	9	27

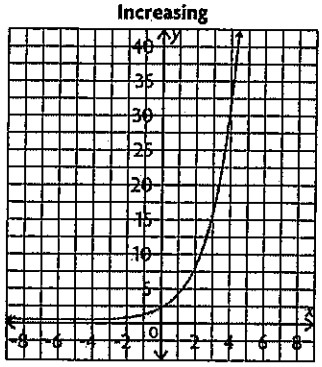
I noticed that the difference between consecutive y-values is not constant. The function is not linear.

I noticed each y-value is $\frac{1}{4}$ the previous y-value as x increases by 1. The function must be an exponential function with a base value of $\frac{1}{4}$ being raised to an exponent and then multiplied by some constant term in the equation. Thus, the answer is yes because for each unit increase in x, the value of y is divided by 4.

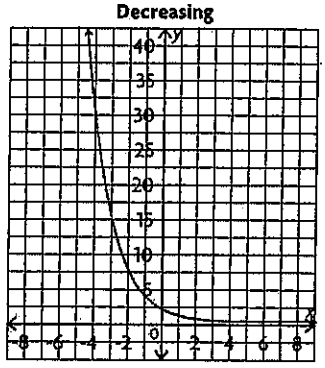
2.a) Number of x-intercepts: 0
y-intercept: $y = 4$; Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
b) Number of x-intercepts: 0
y-intercept: $y = 2$; Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$;
End Behaviour: QII to QI
c) Number of x-intercepts: 0; y-intercept: $y = 7$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
d) Number of x-intercepts: 0; y-intercept: $y = 3$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

3. e.g. Increasing exponential functions increase as x increases, whereas decreasing exponential functions decrease as x increases.

Increasing



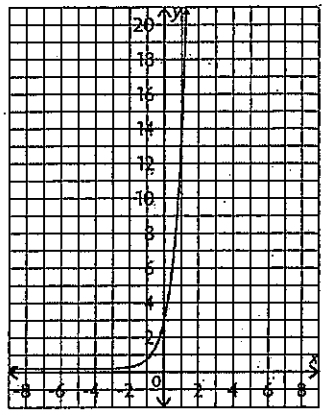
Decreasing



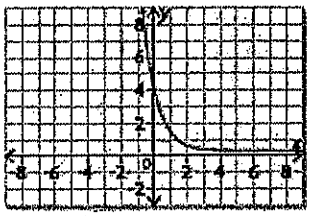
4. a) The y-intercept is 5 and the function is increasing.
$y = 5(2)^x$
$y = 5(2)^{(0)}$
$y = 5(1)$
$y = 5$
The function is increasing because the base is greater than 1.
b) The y-intercept is 2 and the function is decreasing.
$y = 2(0.5)^x$
$y = 2(0.5)^{(0)}$
$y = 2(1)$
$y = 2$
The function is decreasing because the base is less than 1.
c) The y-intercept is 10 and the function is increasing.
$y = 10(1.5)^x$
$y = 10(1.5)^{(0)}$
$y = 10(1.5)$
$y = 10$
The function is increasing because the base is greater than 1.
d) The y-intercept is 1 and the function is decreasing.
$y = (0.4)^x$
$y = (0.4)^{(0)}$
$y = 1$
The function is decreasing because the base is less than 1.

5. a) i) Yes, an exponential equation can be used to model the function because the rate of change in y-values doubles for each unit increase in x.
ii) y-intercept: $y = 1$, the function is increasing.
b) i) No, y increases by 2 as x increases by 1.
ii) y-intercept: $y = 3$, the function is increasing.
c) i) Yes, an exponential equation can be used to model the function because the rate of change in y-values get divided by 4 for each unit increase in x.
ii) y-intercept: $y = 64$, the function is decreasing.
d) i) No. e.g., y decreases, then increases, then decreases again.
ii) y-intercept: $y = 1$, the function is first decreasing than increasing as it reaches the y-intercept.

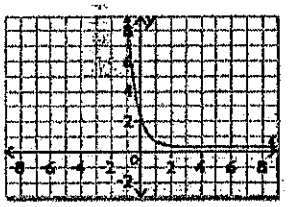
6. a) Number of x-intercepts: 0
y-intercept: $y = 3$; Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



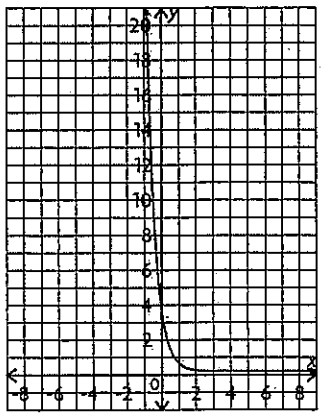
b) Number of x-intercepts: 0; y-intercept: $y = 4$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



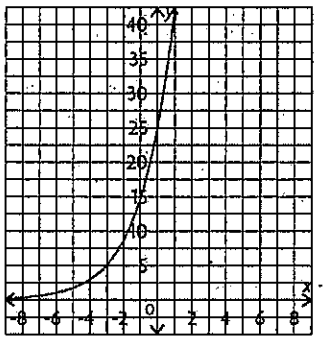
c) Number of x-intercepts: 0; y-intercept: $y = 2$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



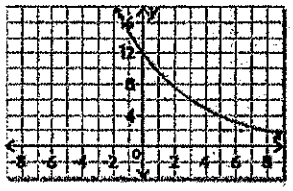
d) Number of x-intercepts: 0; y-intercept: $y = 3.5$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



e) Number of x-intercepts: 0; y-intercept: $y = 25$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



f) Number of x-intercepts: 0; y-intercept: $y = 12$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

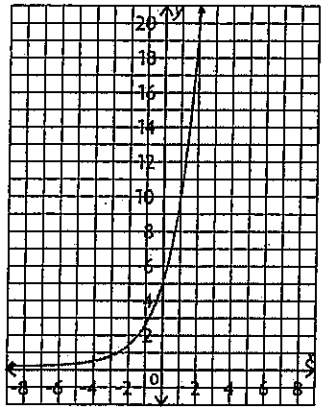


7. a) The y-intercept is positive and since the base is 8, it must mean the function is increasing because the base is larger than 1.
b) The y-intercept is positive and since the base is 0.6, it must mean the function is decreasing because the base is less than 1.
c) The y-intercept is positive and since the base is e, it must mean the function is increasing because e is greater than 1.

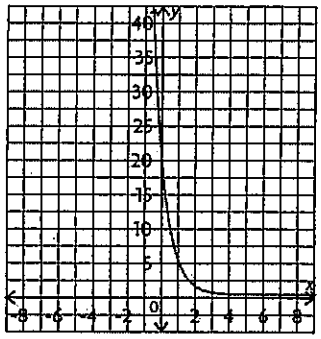
8. a) Number of x-intercepts: 0; y-intercept: $y = 4$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
b) Number of x-intercepts: 0; y-intercept: $y = 8$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
c) Number of x-intercepts: 0; y-intercept: $y = 3$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
d) Number of x-intercepts: 0; y-intercept: $y = 10$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
e) Number of x-intercepts: 0; y-intercept: $y = 30$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
f) Number of x-intercepts: 0; y-intercept: $y = 1$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
g) Number of x-intercepts: 0; y-intercept: $y = 3$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI
h) Number of x-intercepts: 0; y-intercept: $y = 45$
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI

9. a) It is a decreasing exponential function because it is a function with an exponent of x and the base is less than 1.
b) It is not a decreasing exponential function because although it has an exponent of x, the base is greater than 1.
c) It is a decreasing exponential function because it is a function with an exponent of x and the base is less than 1.
d) It is not a decreasing exponential function because although it has an exponent of x, the base is greater than 1.

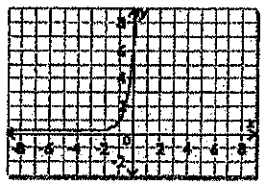
10. a) It is an increasing function because its base is more than 1.



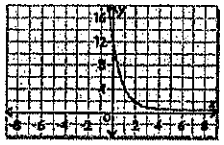
b) It is a decreasing function because its base is less than 1.



c) It is an increasing function because its base is more than 1.

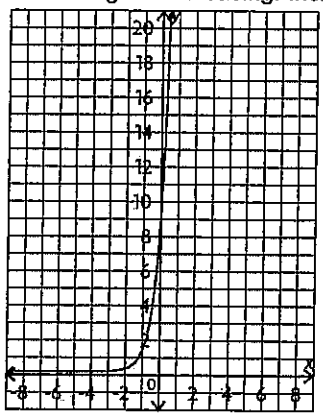


d) It is a decreasing function because its base is less than 1.

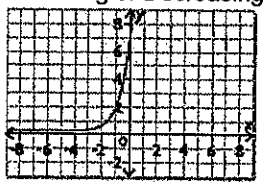


11. a) It is an increasing function with a y-intercept of (0,6). It matches with iii).
b) It is a decreasing function with a y-intercept of (0, 4). It matches with i).
c) It is a decreasing function with a y-intercept of (0, 2). It matches with ii).
d) It is an increasing function with a y-intercept of (0, 3). It matches with iv).

12. a) y-intercept: $y = 9$; Base: 7
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
Increasing or Decreasing: Increasing

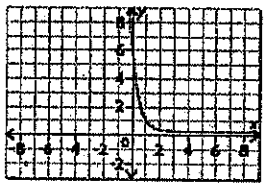


b) y-intercept: $y = 7$; Base: 4
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
Increasing or Decreasing: Increasing

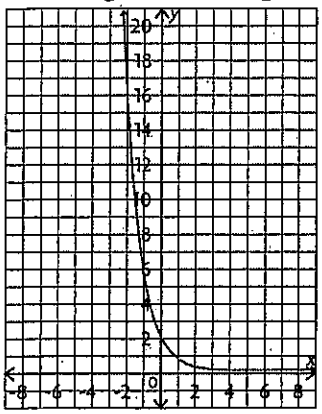


c) y-intercept: $y = 6$; Base: $\frac{1}{7}$

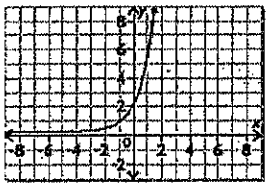
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
Increasing or Decreasing: Decreasing



d) y-intercept: $y = 2$; Base: 0.35
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
Increasing or Decreasing: Decreasing



e) y-intercept: $y = 2$; Base: e
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
Increasing or Decreasing: Increasing



13. a) i) Range: $\{y \mid y > 0, y \in R\}$ a: 2 b: 0.5 This is a decreasing function.
ii) Range: $\{y \mid y > 0, y \in R\}$ a: 1 b: 3
This is an increasing function.
iii) Range: $\{y \mid y > 0, y \in R\}$ a: 3 b: 0.5
This is a decreasing function.
iv) Range: $\{y \mid y > 0, y \in R\}$ a: 2 b: 4
This is an increasing function.

b) i) This function matches with the graph B. because it is a decreasing function with a y-intercept of 2.
ii) This function matches with graph D because it is an increasing function with a y-intercept of 1.
iii) This function matches with the graph A. because it is a decreasing function with a y-intercept of 3.
iv) This function matches with the graph C. because it is an increasing function with a y-intercept of 2.

14. a) e.g. An example of an increasing exponential function with a y-intercept of 5 would be $y = 5(2)^x$ while an example of a decreasing exponential function with a y-intercept of 5 would be $y = 5(0.25)^x$.
b) e.g. Same: number of x-intercepts, y-intercepts, and behaviour, domain, and range. Different: rate of change (increasing vs. decreasing function.

15. Yes, because all exponential functions have the form $y = a(b)^x$. Polynomial functions are characterized by having a numerical exponent while exponential functions have variable exponents.

16.

	$f(x) = ax + b$	$f(x) = a(b)^x$
Number of x-Intercepts	1	0
y-Intercept	b	a
End Behaviour	QIII to QI or QII to QIV	QII to QI
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y \in R\}$	$\{y \mid y > 0, y \in R\}$
Increasing or Decreasing	Either increasing ($a > 0$) or decreasing ($a < 0$)	Either increasing ($b > 1$) or decreasing ($b<1$)

17. The number of x-intercepts, the end behaviour, the domain, and the range are common to all exponential functions. The y-intercept and whether the function increases or decreases are unique to the function.

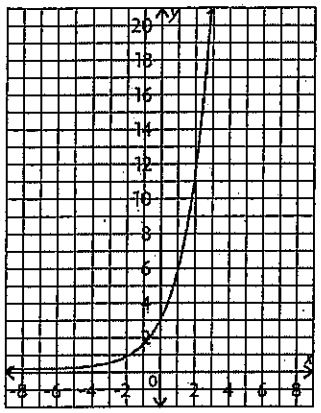
18. a) Student A:
y-intercept: $y = 80$
domain: $\{x \mid x \geq 0, x \in R\}$
range: $\{y \mid 0 < y \leq 8, y \in R\}$
Student B:
y-intercept: $y = 100$
domain: $\{x \mid x \geq 0, x \in R\}$
range: $\{y \mid 0 < y \leq 100, y \in R\}$
b) e.g., Concentration of caffeine in blood naturally decreases over time as the kidneys filter it from the blood into the urine.
c) Student B consumed more caffeine. Student B processed the caffeine more quickly. This can be seen from how much they initially started with and how much they consumed in four hours. Student A had 80 mg of caffeine at the start and consumed 60 mg after four hours. Student B started with 100 mg of caffeine and within the same time period, had consumed 80 mg.
d) Both Student A and B had about 20 mg of caffeine in their body after four hours.
e) The y-Intercepts are the values representing the initial amount of caffeine in the students' bodies. The values are different because the students drank different amounts of the same drink, or they drank different drinks that did not have the same concentration of caffeine.

19. a)

Hour	Students who are Told
0	3
1	6
2	12
3	24
4	48
5	96

b) Yes. e.g., After each hour, the number of students who are told in that hour about the presentation doubles.
c) $y = 3(2)^x$; domain: $\{x \mid x \geq 0, x \in R\}$
range: $\{y \mid y \geq 3, y \in R\}$

d) After 8 h, 768 students would be told.



20. a) Yes. e.g., Any data with a constant doubling time can be expressed with an exponential function.
b) $y = 4(1.26)^x$; *a* represents the initial number of requests, *b* represents the rate of growth of the number of requests, *x* represents the amount of time in hours since the news broke, *y* represents the total number of interview requests.
c) domain: $\{x \mid x > 0, x \in N\}$
range: $\{y \mid y > 4, y \in N\}$
d)

Time	x	y
9:00 a.m.	0	4.0
10:00 a.m.	1	5.0
11:00 a.m.	2	6.4
12:00 p.m.	3	8.0
1:00 p.m.	4	10.1
2:00 p.m.	5	12.7
3:00 p.m.	6	16.0
4:00 p.m.	7	20.2

Lesson 6.3: Solving Exponential Equations, page 361

1. a) When $y = 3$, $x = 2.00$
When $y = 15$, $x = -0.3$
b) When $y = 3$, $x = -2.8$
When $y = 15$, $x = 6.0$

2. a) $8^x = 64$
$8^x = 8^2$
$x = 2$

LS	RS
8^2	64
64	

LS = RS

b) $2^t = 1$
$2^t = 2^0$
$t = 0$

LS	RS
2^0	1
1	

LS = RS

c) $13^n = \frac{1}{169}$
$13^n = \frac{1}{13^2}$
$13^n = 13^{-2}$
$n = -2$

LS	RS
13^{-2}	$\frac{1}{169}$
$\frac{1}{13^2}$	
$\frac{1}{169}$	

LS = RS

$3^x = \sqrt{27}$
$3^x = \sqrt{3^3}$
d) $3^x = 3^{3\left(\frac{1}{2}\right)}$
$x = \frac{3}{2}$

LS	RS
$3^{\frac{3}{2}}$	$\sqrt{27}$
$(3^3)^{\frac{1}{2}}$	
$27^{\frac{1}{2}}$	
$\sqrt{27}$	

LS = RS

3. a) This is an increasing function. e.g. the base is greater than 1.
b) *x* represents the generations before Joshua, *y* represents the total number of ancestors (e.g., grandparents) Joshua has at generation *x*
domain:
Domain: $\{x \mid x \in N\}$; Range: $\{y \mid y \in N\}$

c) $y = 1(2)^7$
$y = 1(128)$
$y = 128$
Joshua had 128 ancestors seven generations ago.
d) $4096 = 1(2)^x$
$4096 = (2)^x$
When $x = 10$, $2^x = 1024$
When $x = 11$, $2^x = 2048$
When $x = 12$, $2^x = 4096$
There were 4096 ancestors 12 generations ago.

4. a) $10^x = 1000$
$10^x = 10^3$
$x = 3$

b) $2^{-t} = 32$
$2^{-t} = 2^5$
$-t = -5$
$t = -5$

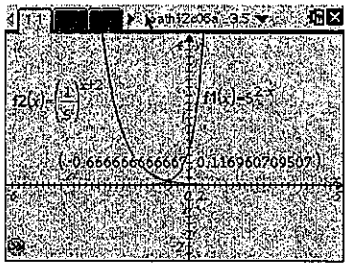
c) $3^n = \frac{1}{81}$
$3^n = \frac{1}{3^4}$
$3^n = 3^{-4}$
$n = -4$

d) $3^{3x+1} = 81$
$3^{3x+1} = 3^4$
$3x + 1 = 4$
$3x = 3$
$x = 1$

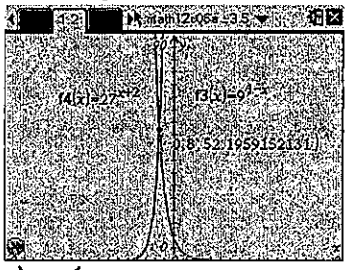
e) $2^{x+2} = (8)^{\frac{1}{2}}$
$2^{x+2} = (2^3)^{\frac{1}{2}}$
$2^{x+2} = 2^{\frac{3}{2}}$
$x + 2 = \frac{3}{2}$
$x = \frac{3}{2} - \frac{4}{2}$
$x = -\frac{1}{2}$

f) $4^{3x} = \frac{1}{16}$
$4^{3x} = 4^{-2}$
$3x = -2$
$x = \frac{2}{3}$

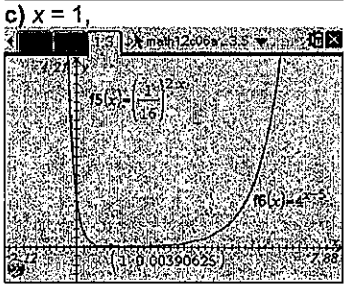
5. a) $x = -\frac{3}{2}$



b) $x = -\frac{4}{5}$



c) $x = 1$,



6. a) e.g., Using a common base of 8 for both sides:
$8^{3x-2} = 64^{x+1}$
$8^{3x-2} = (8^2)^{x+1}$
$8^{3x-2} = 8^{2x+2}$
$3x - 2 = 2x + 2$
$x = 4$

b) e.g., Using a common base of 2 for both sides:
$8^{3x-2} = 64^{x+1}$
$(2^3)^{3x-2} = (2^6)^{x+1}$
$9^{9x-6} = 2^{6x+6}$
$9x - 6 = 6x + 6$
$3x = 12$
$x = 4$
Yes, you can because the answer is the same one as in part a).

7. a) $3^{x+1} = 9$

$3^{x+1} = 3^2$

$x + 1 = 2$

$x = 1$

$8(3)^{\frac{x}{2}} = 72$

$8(3)^{\frac{x}{2}} = 8(9)$

b) $3^{\frac{x}{2}} = 3^2$

$\frac{x}{2} = 2$

$x = 4$

c) $5^{3x+2} = 5$

$3x + 2 = 1$

$3x = -1$

$x = -\frac{1}{3}$

d) $\sqrt{8} = 2^{4x+1}$

$8^{\frac{1}{2}} = 2^{4x+1}$

$(2^3)^{\frac{1}{2}} = 2^{4x+1}$

$2^{\frac{3}{2}} = 2^{4x+1}$

$\frac{3}{2} = 4x + 1$

$\frac{3}{2} - \frac{2}{2} = 4x$

$\frac{1}{2} = 4x$

$\frac{1}{8} = x$

e) $4^{1-2x} = 32$

$(2^2)^{1-2x} = 2^5$

$2^{2-4x} = 2^5$

$2 - 4x = 5$

$-4x = 3$

$x = -\frac{3}{4}$

f) $10^{2(x-3)} = 1000$

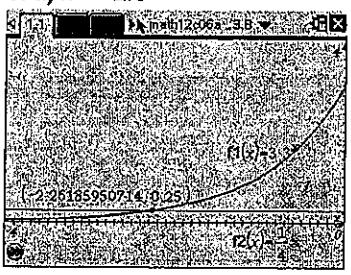
$10^{2(x-3)} = 10^3$

$2x - 6 = 3$

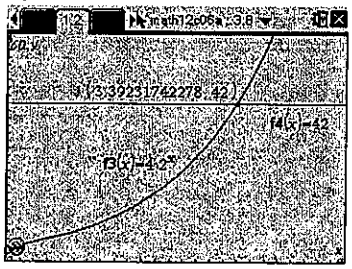
$2x = 9$

$x = \frac{9}{2}$

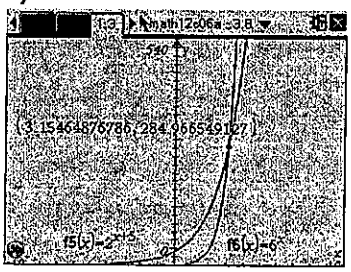
8. a) $x = -2.3$



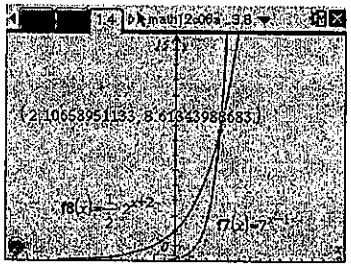
b) $x = 3.4$



c) $x = 3.2$

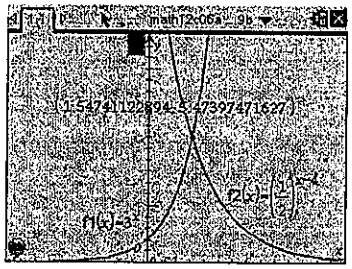


d) $x = 2.1$



9. a) e.g., Graph each function and identify the intersection point.

b) (1.5, 5.5)



10. a) $50 = 100\left(\frac{1}{2}\right)^{\frac{t}{5.3}}$

$$\frac{50}{100} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

$$\frac{1}{2} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$$

$$1 = \frac{t}{5.3}$$

$t = 5.3$ years

It takes 5.3 years.

b) $A(t) = 100\left(\frac{1}{2}\right)^{\frac{10}{5.3}}$

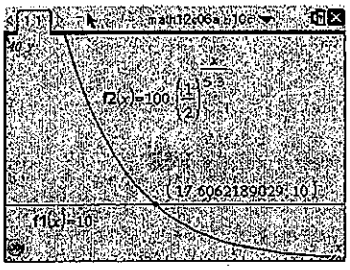
$A(t) = 100\left(\frac{1}{2}\right)^{1.886...}$

$A(t) = 100(0.270...)$

$A(t) = 27.040...\%$

27% will remain in a sample after 10 years.

c) t = 17.6 years,



11. a) $P(t) = 12\,000(3)^{\frac{t}{4}}$

$$324\,000 = 12\,000(3)^{\frac{t}{4}}$$

$$12\,000(27) = 12\,000(3)^{\frac{t}{4}}$$

$$12\,000(3)^3 = 12\,000(3)^{\frac{t}{4}}$$

$$3 = \frac{t}{4}$$

t = 12 hours

The value of t is 12 hours.

b) The number of bacteria in a Petri dish after 12 h is 324 000.

c) Using graphing technology, the answer is t = 0.56 h.

12. a) $H(t) = 130\left(\frac{1}{2}\right)^{\frac{t}{43.3}}$

$H(t) = 130\left(\frac{1}{2}\right)^{\frac{8}{43.3}}$

$H(t) = 12.009...$

The cheese will keep for 12 h.

b) Number of hours: (5 d)(24 h/d) = 120 h

Using graphing technology, it should be kept at 5.0 °C

13. a) The initial price of the couch was $2500.

e.g., It is the y-intercept of the function.

b) The couch cost less than half of its initial price after the fourth week. This was determined by setting $C(t)$ equal to 1250 and solve for t; then round up.

c) $655.36 = 2500(0.8)^t$

$$\frac{655.36}{2500} = (0.8)^t$$

$$\frac{65\,536}{250\,000} = (0.8)^t$$

$$\frac{16\,384}{62\,500} = (0.8)^t$$

$$\frac{4096}{15\,625} = (0.8)^t$$

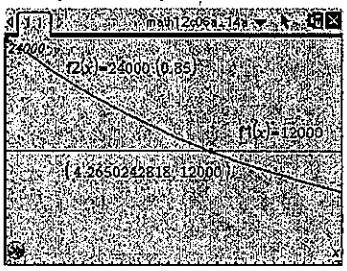
$$\frac{4^6}{5^6} = (0.8)^t$$

$$\left(\frac{4}{5}\right)^6 = \left(\frac{4}{5}\right)^t$$

t = 6 weeks

Eric bought the couch in the sixth week.

14. a) t = 4.27 years



b) Yes, it is possible, if a specific purchase date was assumed in 2002. If the vehicle was assumed to be purchased on January 1, 2002, then 4 years from that date would be December 31, 2005. Four months or 0.3 years further from this date would bring the date into 2006. Therefore, at some point in 2006, the vehicle will be worth $12 000.

15. $307\,200 = 300(2)^{\frac{h}{12}}$

$$1024 = 2^{\frac{h}{12}}$$

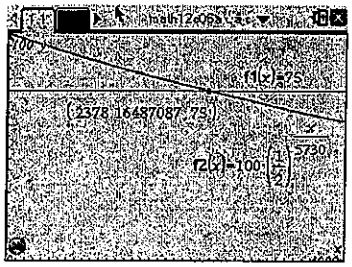
$$2^{10} = 2^{\frac{h}{12}}$$

$$10 = \frac{h}{12}$$

$$h = 120 \text{ h}$$

It will take 120 h for the bacterial population to grow to 307 200.

16. a) This artifact is 2378 years old.



b) $25 = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

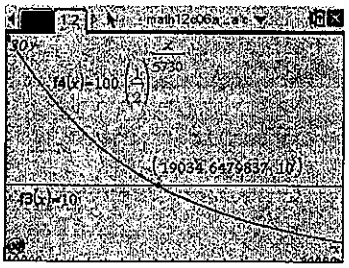
$$\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$2 = \frac{t}{5730}$$

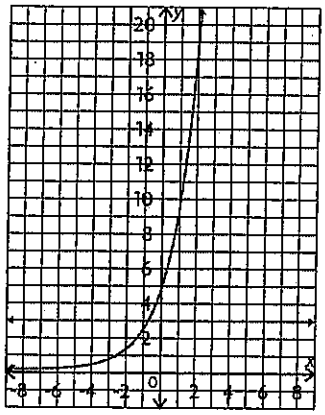
$$t = 11\,460 \text{ years}$$

This artifact is 11 460 years old.

c) This artifact is 19 035 years old.



17. e.g., Graphing systems of equations to determine an intersection is like solving exponential equations algebraically because we are trying to locate a point when x and y are equal for both equations. In the graphs, $y = 3$ and $y = 5(2)^x$ are equal at their intersection point.



18. $2(4)^{2x} = \frac{1}{32}$

$$128(4)^{2x} = 2$$

$$2^7(2^2)^{2x} = 2$$

$$2^7(2^{4x}) = 2$$

$$2^{4x+7} = 2$$

$$4x + 7 = 1$$

$$4x = -6$$

$$x = -\frac{3}{2}$$

Mid-Chapter Review, page 368

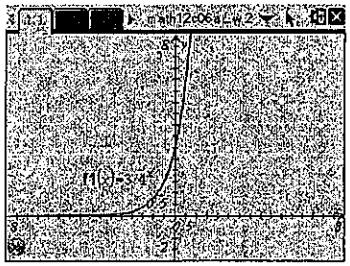
1. a) This graph does not represent an exponential function because it does not have a horizontal asymptote. In addition, its end behaviour is unlike that of an exponential equation since it extends from QII to QIV.

b) This graph does represent an exponential function because it has all of the basic characteristics of an exponential function. It is constantly decreasing at an unsteady rate like an exponential decay function. It has a y-intercept but no x-intercept. It has a domain of $\{x \mid x \in R\}$ and $a > 0$ and $0 < b < 1$. Also, it decreases from QII to QI.

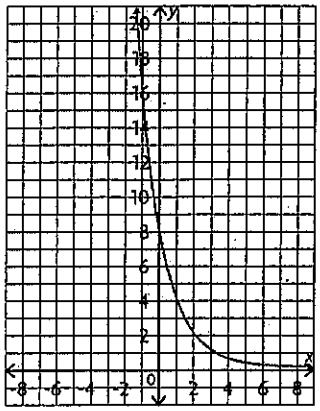
c) This graph does represent an exponential function because it has all of the basic characteristics of an exponential function. It is constantly increasing at an unsteady rate like an exponential growth function. It has a y-intercept but no x-intercept. It has a domain of $\{x \mid x \in R\}$ with $a < 0$ and $0 < b < 1$. Also, it decreases from QII to QI.

d) This graph does represent an exponential function because it has all of the basic characteristics of an exponential function It is constantly decreasing at an unsteady rate like an exponential decay function. It has a y-intercept but no x-intercept. It has a domain of $\{x \mid x \in R\}$, with $a < 0$ and $b > 1$. Also, it decreases from QII to QI.

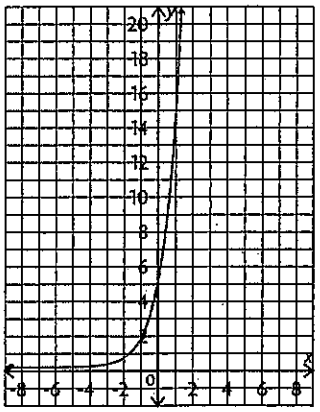
2. a) i)



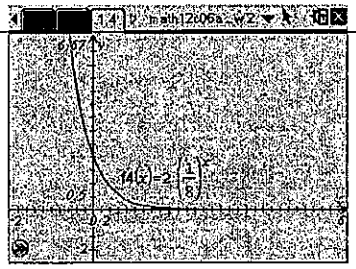
ii)



iii)

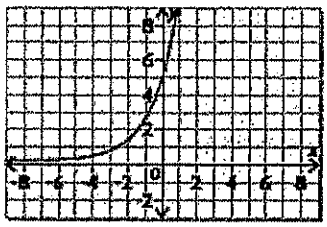


iv)

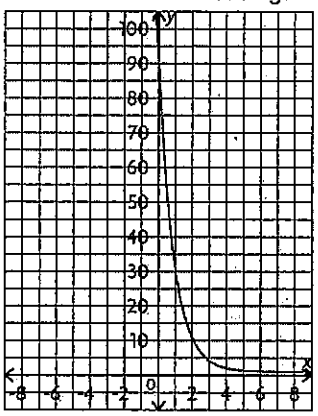


b) i) number of x intercepts: 0; y-intercept: $y = 3$
domain: $\{x \mid x \in R\}$; range: $\{y \mid y < 0, y \in R\}$
end behaviour: QII to QI
ii) number of x-intercepts: 0; y-intercept: $y = 8$
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
end behaviour: QII to QI
iii) number of x-intercepts: 0; y-intercept: $y = 5$
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
end behaviour: QII to QI
iv) number of x-intercepts: 0; y-intercept: $y = 2$
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
end behaviour: QII to QI

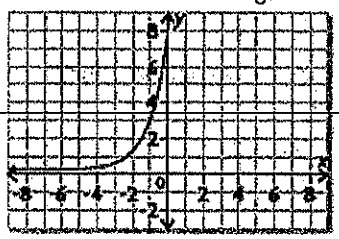
3. a) Number of x-intercepts: 0; y-intercept: $y = 5$
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
The function is increasing.



b) Number of x-intercepts: 0; y-intercept: $y = 90$
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
The function is decreasing.

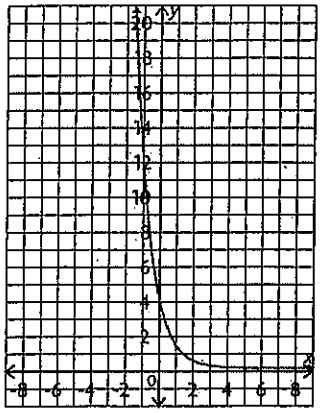


c) number of x-intercepts: 0
y-intercept: 8
domain: $\{x \mid x \in R\}$
range: $\{y \mid y > 0, y \in R\}$
The function is increasing.

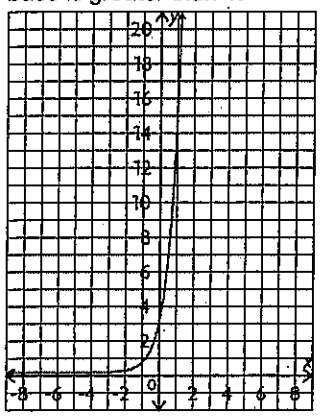


4. a) b) $a = 2$; c) $a = 5$; d) $a = 3$
b) b) It is between $0 < b < 1$ because the function is decreasing.
c) It is between $0 < b < 1$ because the function is decreasing.
d) b is greater than 1 because the function is increasing.
c) b) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
c) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
d) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$

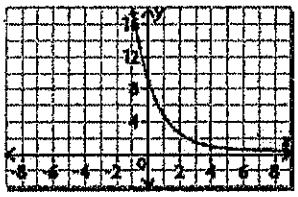
5. a) e.g., It is a decreasing function because the base is less than 1.



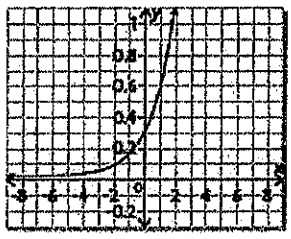
b) e.g., It is an increasing function because the base is greater than 1.



c) e.g., It is a decreasing function because the base is less than 1.



d) e.g., It is an increasing function because the base is greater than 1.



6. a) Assuming that Lauren is not allowed to vote, the following is a table of values that models this situation:

Distribution Cycles After Council Votes (x)	Number of Students who received Handouts (y)
0	7
1	49
2	343
3	2401
4	16807

b) Yes. $y = 7(7)^x$; e.g., The ratio of consecutive numbers of students polled in each round is constant.

7. a) $3^{2x} = 243$

$3^{2x} = 3^5$

$2x = 5$

$x = \frac{5}{2}$

LS	RS
$3^{2\left(\frac{5}{2}\right)}$	243
3^5	
243	

LS = RS

b) $5^x = \frac{1}{125}$

$5^x = \frac{1}{5^3}$

$5^x = 5^{-3}$

$x = -3$

LS	RS
5^{-3}	$\frac{1}{125}$
$\frac{1}{5^3}$	
$\frac{1}{125}$	

LS = RS

c) $4^{x+2} = 64$

$4^{x+2} = 4^3$

$x + 2 = 3$

$x = 1$

LS	RS
4^{-1+2}	$\sqrt{16}$
4^1	4
4	

LS = RS

d) $6^{2x+1} = \frac{1}{216}$

$6^{2x+1} = \frac{1}{6^3}$

$6^{2x+1} = 6^{-3}$

$2x + 1 = -3$

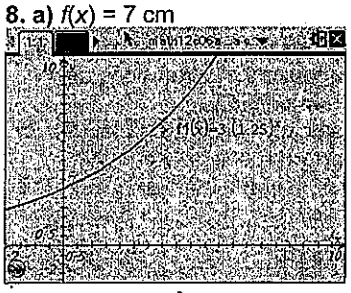
$2x = -2$

$x = -1$

LS	RS
$6^{2(-1)+1}$	$\frac{1}{126}$
6^{-3}	
$\frac{1}{6^3}$	
$\frac{1}{126}$	

LS = RS

8. a) $f(x)$ = 7 cm



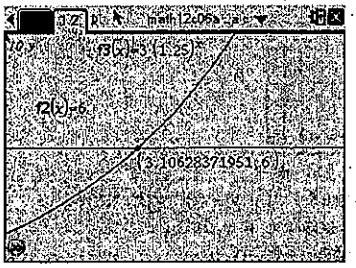
b) $f(0) = 3(1.25)^0$

$f(0) = 3(1)$

$f(0) = 3$ m

The initial height of the plant was 3 cm.

c) x = 3.1 months



The plant was double its original height during the 4th month.

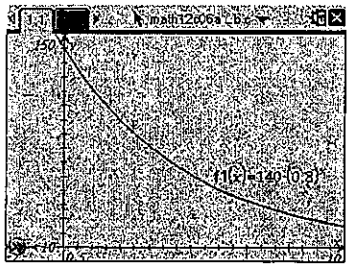
9. a) $y = 140(0.80)^0$

$y = 140(1)$

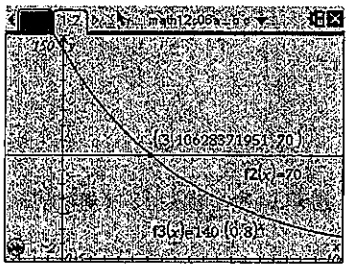
$y = 140$ cm

The ball was first dropped from a height of 140 cm.

b) y = 37 cm



c) x = 3.106... when y = 70 cm



The height was less than half the initial drop height on the 4th bounce.

10. a) The initial temperature of the coffee is 90 °C. This is because it is the y-intercept.

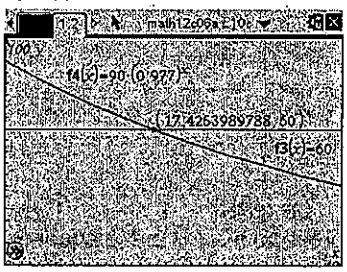
b) $C(t) = 90(0.977)^t$

$C(t) = 90(0.977)^5$

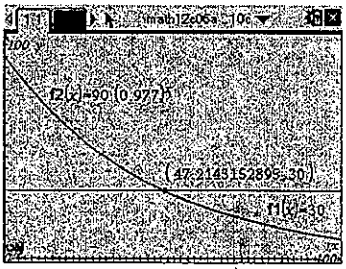
$C(t) = 80.115...$

The temperature of the coffee after five minutes is 80 °C.

c) $C(t)$ = 60 °C when t = 17 minutes.



$C(t)$ = 30 °C when t = 47 minutes.



11. a) I decided to express both sides of the equation with the same base.

$$3(2)^{4x} = 192$$
$$2^{4x} = 64$$
$$2^{4x} = 2^6$$
$$4x = 6$$
$$x = \frac{3}{2}$$

LS	RS
$3(2)^{4\left(\frac{3}{2}\right)}$	192
$3(2)^6$	
$3(64)$	
192	

LS = RS

b) I decided to express both sides of the equation with the same base.

$$3^{2-x} = \frac{1}{243}$$
$$3^{2-x} = (243)^{-1}$$
$$3^{2-x} = (3^5)^{-1}$$
$$3^{2-x} = 3^{-5}$$
$$2 - x = -5$$
$$-x = -7$$
$$x = 7$$

LS	RS
$3^{(2-7)}$	$\frac{1}{243}$
3^{-5}	
$\frac{1}{3^5}$	
$\frac{1}{243}$	

LS = RS

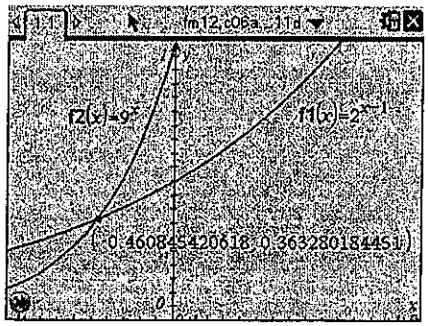
c) I decided to express both sides of the equation with the same base.

$$4^{2x} = 8^{x+1}$$
$$2^{2(2x)} = 2^{3(x+1)}$$
$$2^{4x} = 2^{3x+3}$$
$$4x = 3x + 3$$
$$4x - 3x = 3$$
$$x = 3$$

LS	RS
4^{2x}	8^{x+1}
$4^{2(3)}$	$8^{(3)+1}$
4^6	8^4
4096	4096

LS = RS

d) Using a graphing calculator, x is –0.46.



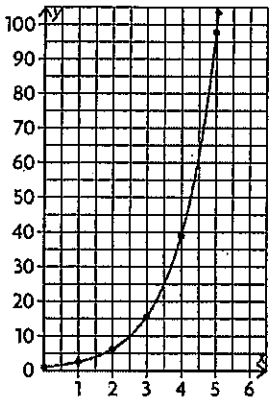
LS	RS
2^{x-1}	9^x
$2^{-0.46-1}$	$9^{-0.46}$
$2^{-1.46}$	0
0.363...	0.363...

LS = RS

Lesson 6.4: Modelling Data Using Exponential Functions, page 377

1. a) This data set does not involve exponential growth or decay because the differences are constant. It is a linear function.

b) This data set does involve exponential growth because it has constant ratios between the y-values with each consequent unit value of x. The function is $y = (2.5)^x$.



c) This data set does not involve exponential growth or decay because the differences are not constant, or a constant ratio.

d) This data set does not involve exponential growth or decay the differences are not constant, or a constant ratio.

2. a) Using a graphing calculator, the exponential regression function for the data is $y = 10.097...(0.200...)^x$.

Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$

y-Intercept: $y = 10.097...$;

End Behaviour: QII to QI

This function is decreasing and shows exponential decay.

b) Using a graphing calculator, the exponential regression function for the data is $y = 2.780...(1.054...)^x$.
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
y-Intercept: $y = 2.780...$; End Behaviour: QII to QI
This function is increasing and shows exponential growth.

3. a)

Years since Retirement	Rent ($)	Ratios
0	9600	
1	9960	1.037...
2	10 344	1.038...
3	10 752	1.039...

Yes, an exponential model can represent the data since the ratio of consecutive pairs of y-values are close.
b) The equation will be of the form $f(x) = a(b)^x$. Thus, We can create the formula by setting a to the first rent value and b to the average ratio of consecutive pairs of y-values. x will be the number of years since retirement. $f(x)$ will be the rent after x years.
c) Using graphing technology, the regression equation is $y = 9595.433...(1.038...)^x$.

$f(x) = 9595.433...(1.038...)^x$

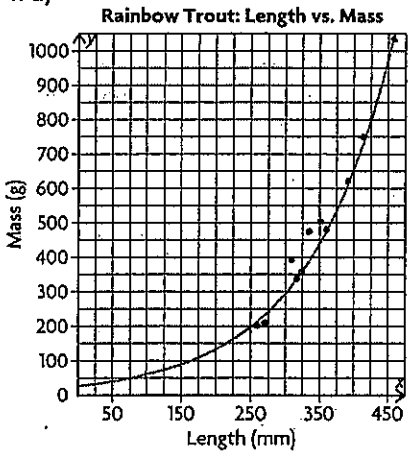
$f(x) = 9595.433...(1.038...)^{10}$

$f(x) = 9595.433...(1.459...)$

$f(x) = 14\ 000.638...$

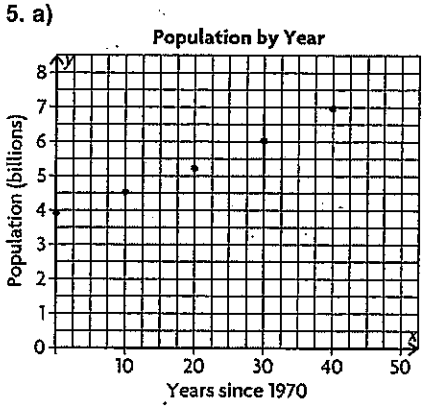
I predict the Belands will pay about $14 000 in 10 years.

4. a)



b) Using a graphing calculator, the exponential regression function for the data is $y = 26.934...(1.008...)^x$.
c) e.g., 690 g; I identified the point on the curve that had an x-value of 400.
d) I would expect this fish to be about 446 mm long. I identified the point on the curve that had an y-value of 1000.

5. a)



b) Using a graphing calculator, the exponential regression function for the data is $y = 3.911...(1.014...)^x$.
c) The population is estimated to be 8.05 billion. To get this number, 50 (for 2020) was substituted for x in the regression equation so that y could be determined.
d) The population is expected to reach 9.50 billion 61.5 years after 1970, that is, during the year 2031. I plotted $y = 9.50$ and the regression function on a graphing calculator. The x-coordinate of the point at which these two functions intersect is the point at which the population is expected to reach 9.50 billion.

6. a) Using a graphing calculator, the exponential regression equation for the data is $y = 14.429...(1.065...)^x$.

b) $y = 14.429...(1.065...)^{10}$

$y = 14.429...(1.879...)$

$y = 27.119...$ cm

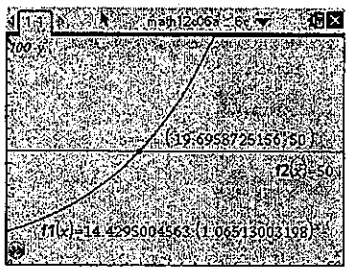
The height of the sunflower on day 10 was about 27.1 cm.

c) $y = 14.429...(1.065...)^{30}$

$y = 14.429...(6.638...)$

$y = 95.792...$ cm

The height of the sunflower on day 30 was about 95.8 cm. The answer does not make sense since the height was 98.0 cm on day 28.
d) I would expect the sunflower to reach a height of 50 cm on day 20.



7. a) Using graphing technology, I input the x and y values into the spreadsheet. I then used the exponential regression function to determine the equation that models this growth. The equation is $y = 2^x$.

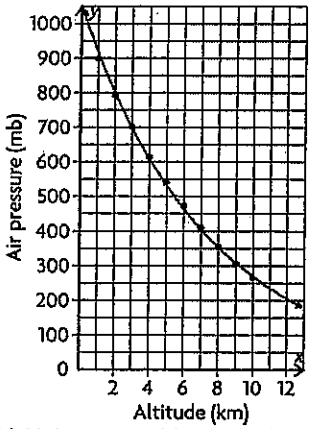
b) Under this context, the domain is $\{x \mid x \in N\}$, and the range is $\{y \mid y > 0, y \in N\}$.

c) If we extend the first and last column of the table, we get the following:

Number of Digits	Number of Possible Codes
1	2
2	4
3	8
4	16
5	32
6	64
7	128

Based on the table, 104 codes lies in between the codes for 6 digits and the codes for 7 digits. 6 digits would not provide enough codes. Therefore, 7 digits would be needed to have 104 codes.

8. a) Here is the graph.



b) Using a graphing calculator, the exponential regression function for the data is $y = 1050.311...(0.873...)^x$.

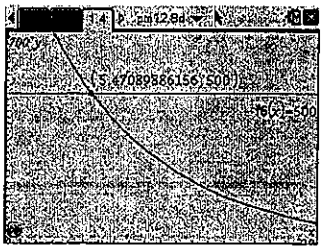
c) $y = 1050.311...(0.873...)^{15}$

$y = 1050.311...(0.130...)$

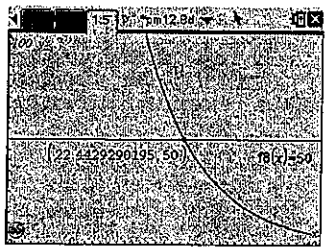
$y = 137.249...$ mb

The pressure at 15 km is about 137.3 mb.

d) Using graphing technology, the pressure reaches 500.0 mb at about 5 km.



Using graphing technology, the pressure reaches 50.0 mb at about 22 km.



9. a)

Time (years)	Frog Population	Decrease in Population (%)
0	575	
1	460	20%
2	368	20%
3	294	20.1%
4	235	20.1%
5	188	20%

Average Decrease in Population = 20.04%

Therefore, the frog population is decreasing by 20% every year.

b) Using a graphing calculator, the exponential regression function for the data is $y = 575.228...(0.799...)^x$.

c) e.g., Half the population is $\frac{575}{2}$ or about 288 frogs. One possible estimate could be that the half-life is 3 years, since the size of the population at 3 years is 294, almost equal to half the initial population. Using the regression equation, we determine that the half-life is about 3.1 years.

10. a)

Year	Atmospheric CO_2 (ppm)	First Differences
1960	319	
1965	322	3
1970	328	6
1975	333	5
1980	341	8
1985	348	7
1990	356	8
1995	364	8
2000	372	8
2005	381	9

e.g., No, the rate of change of atmospheric CO_2 is increasing.

b) x: The number of years since 1960.

y: The atmospheric CO_2; $a = 319$

Solve for b using the a-value and one of the points in the table. Using (45, 381):

$381 = 319b^{45}$

$1.194... = b^{2005}$

$b = 1.003...$

Therefore, the exponential function is $y = 319(1.003...)^x$.

c) Using a graphing calculator, the exponential regression function for the data is $y = 315.609...(1.004...)^x$.

d) Predicted Proportion in 2010

$y = 315.609...(1.004...)^{50}$

$y = 315.609...(1.225...)$

$y = 386.609...$ cm

The predicted proportion in 2010 is 387 ppm.

Predicted Proportion in 2020

$y = 315.609...(1.004...)^{60}$

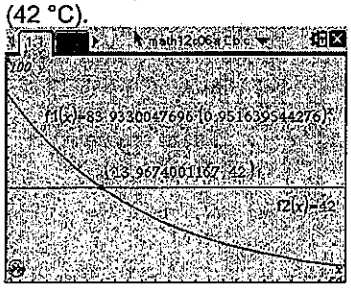
$y = 315.609...(1.275...)$

$y = 402.687...$ cm

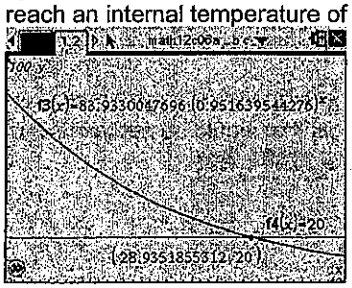
The predicted proportion in 2020 is 403 ppm.

11. a) Using a graphing calculator, the exponential regression function for the data is $y = 83.933...(0.951...)^x$.

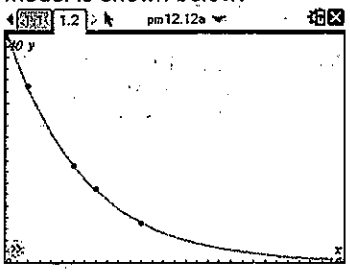
b) It will take the sandwich 14 minutes to have an internal temperature of half its initial temperature (42 °C).



c) The sandwich will need to sit for 29 minutes to reach an internal temperature of 20 °C.

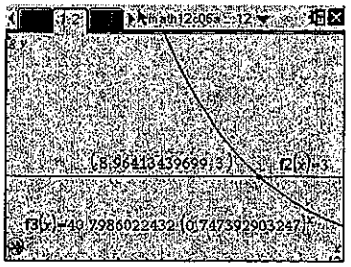


12. a) Using graphing technology to determine the regression function, a graphical exponential model is shown below.

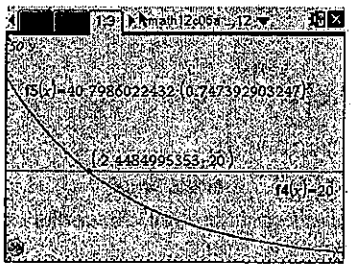


A function that models this situation algebraically is $y = 40.799...(0.747...)^x$.

b) Tristan will have less than 3 mg of caffeine in his body after 9 hours.



c) Half the initial amount of caffeine in Tristan's body is 20 mg. The half-life of caffeine in Tristan's body is about 2 h.



13. a) Using graphing technology, the equation is $y = 4120.075...(1.499...)^x$.

b) $y = 4120.075...(1.499...)^{10}$

$y = 4120.075...(57.659...)$

$y = 237\ 559.646...$

There will be 237 560 bacteria.

c) Assuming that the observations are made every hour, the function that models the growth of these bacteria is $y = 4120.075...(1.2)^x$. The base has changed to 1.2 because the bacteria are growing at a rate of 20% per hour. Since the population is growing, the value must be greater than 1.

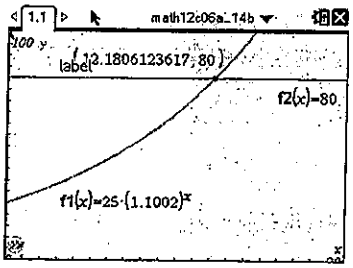
14. a)

Week	Pond Surface Covered by Algae (%)	Growth (%)
0	25.0	
1	27.5	10.0
2	30.3	10.2
3	33.3	9.9
4	36.6	9.9
5	40.3	10.1

$$\frac{(10.0+10.2+9.9+9.9+10.1)}{5} = 10.02\%$$

The algae are growing at a rate of about 10% every week.

b) Use the equation $y = 25(1.100...)^x$ and a graphing calculator to solve this.



It will take 12.2 weeks for the algae to cover 80% of the pond.

15. a) i) $P(26) = 200\left(\frac{1}{2}\right)^{\frac{26}{48}}$

$P(26) = 200\left(\frac{1}{2}\right)^{0.541...}$

$P(26) = 200(0.686...)$

$P(26) = 137.395...$ barrels/week

The expected production of this well, 26 weeks after production begins to drop is 137 barrels/week.

ii) $P(26) = 200\left(\frac{1}{2}\right)^{\frac{100}{48}}$

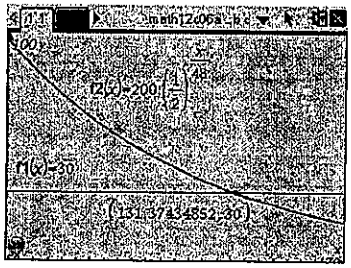
$P(26) = 200\left(\frac{1}{2}\right)^{2.083...}$

$P(26) = 200(0.235...)$

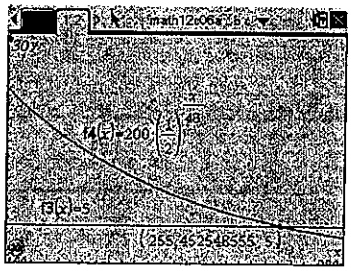
$P(26) = 47.193...$ barrels/week

The expected production of this well, 100 weeks after production begins to drop is 47 barrels/week.

b) Production will reach this level during the 132nd week.



c) The company should cap the well during the 256th week.



16. a) $y = 3000(0.9)^x$, where x is the number of weeks that the well has operated for and y is the weekly amount of oil produced, in barrels.

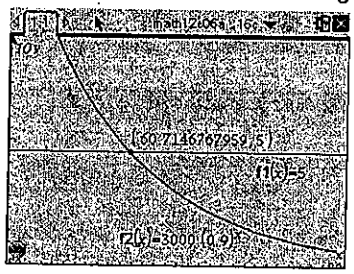
b) $y = 3000(0.9)^{20}$

$y = 3000(0.121...)$

$y = 364.729...$

At the end of 20 weeks of operation, 365 barrels of oil will be extracted from the well.

c) The well will be closed during the 61st week.



17. a) e.g., Calculate the rate of change in the decrease (b) and use 100 for a. Use software or a calculator to perform exponential regression to get those values.

b) Using the method from part a) to create an algebraic model, we get our equation as so:

$97 = 100b^1$

$b = 0.97$

$y = 100(0.97)^x$

Now substitute 7 for x:

$y = 100(0.97)^7$

$y = 100(0.807...)$

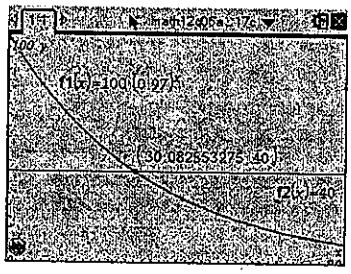
$y = 80.798...\%$

The light intensity would be 80.80%.

c) $100 - 60 = 40$

The light intensity has to go below 40%.

The minimum of gels that would be needed is 31.



18. a) Perimeter: 4(4) = 16; Inner Rows: 4(3) = 12 Inner Columns: 4(3) = 12; 16 + 12 + 12 = 40

40 toothpicks will be needed to create the next figure in the pattern.

b)

Figure	Number of Toothpicks	1st Diff.	2nd Diff.
1	4		
2	12	8	
3	24	12	4
4	40	16	4

The second differences are constant, so the data is best modelled by a quadratic function.

c) Continuing the table:

Figure	Number of Toothpicks	1st Diff.	2nd Diff.
1	4		
2	12	8	
3	24	12	4
4	40	16	4
5	60	20	4
6	84	24	4
7	112	28	4

112 toothpicks will be needed to make the 7th figure.

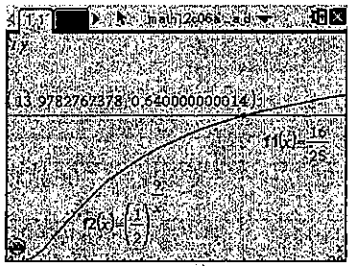
19. a) Using point (9, 64.0), calculate h as shown:

$$64.0 = 100.0\left(\frac{1}{2}\right)^{\frac{9}{h}}$$

$$\frac{64}{100} = \left(\frac{1}{2}\right)^{\frac{9}{h}}$$

$$\frac{16}{25} = \left(\frac{1}{2}\right)^{\frac{9}{h}}$$

Use a graphing calculator to solve this equation.



The value of h is 13.978... or 14 days.

b) $A(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{13.978...}}$

c) $a = 100$

Use the same point used to calculate h in a) to calculate b for this part. Using (9, 64.0), b is calculated as shown:

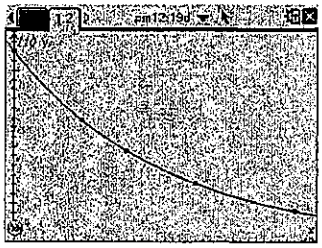
$$64.0 = 100b^9$$

$$\frac{16}{25} = b^9$$

$$b = 0.951...$$

The exponential regression equation is $y = 100(0.951...)^x$.

d) Using graphing technology, the exponential regression equation is $y = 100.001...(0.951...)^x$. It is a good model for the data because the function lies very close to the scatter plot.



The equation determined by hand and the one determined using graphing technology are very close because the difference in numbers start showing up in the thousandth decimal place.

History Connection, page 384

A. For example, uranium, thorium, radon, and radium.

B. The radioactive decay of an element can be modeled by a half-life equation, which is an exponential decay function.

The half-lives of the four elements listed in prompt A is given below.

- U-235: 7.1×10^8 years
- U-238: 4.51×10^9 years
- Rn-220: 55 s
- Rn-222: 3.823 days
- Ra-226: 1600 years
- Th-227: 18.5 days
- Th-230: 8.0×10^4 years
- Th-231: 25.5 h
- Th-234: 24.1 days

To determine the half-life equations for the elements shown above, I substituted each half-life into the half-life equation. $A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{h}}$, where $A(x)$ is the amount of the element at time x and A_0 is the original amount. The half-life equations for the elements are (where x represents time in years, except as noted):

U-235:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{7.1\times10^8}}$$

U-238:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{4.51\times10^9}}$$

Rn-220:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{55}}$$

where x represents time in seconds

Rn-222:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{3.823}}$$

where x represents time in days

Ra-226:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{1600}}$$

Th-227:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{18.5}}$$

where x represents time in days

Th-230:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{8.0\times10^4}}$$

Th-231:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{25.5}}$$

where x represents time in hours

Th-234:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{2.41}}$$

where x represents time in days

Applying Problem-Solving Strategies, page 385

A.–G. Answers will vary, based on the decisions that players make.

H. Possible variations:

- Increase the size of the playing square, or use another shape.
- If a player hits a square, she or he gets to go again immediately.
- Once a function is announced, the opposing player must tell the person if the function passes above, below, or between her or his squares.
- Assign points to each hit, or start out with a given number of points and deduct points for misses or questions (for example, "Does the function pass above your squares?").

Lesson 6.5: Financial Applications Involving Exponential Functions, page 395

1. a) Danielle invested \$1500 in account A and \$2000 in account B.

b) The annual interest rate for investment A was 5%, and the annual interest rate for investment B was 3.5%. You get these rates by subtracting 1 from the b-values (since they are 1 plus the interest rate), and then you multiply by 100%.

c) A. $y = 1500(1.05)^5$

$y = 1500(1.276...)$

$y = \$1914.422...$

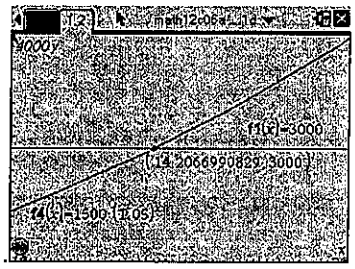
B. $y = 2000(1.035)^5$

$y = 2000(1.187...)$

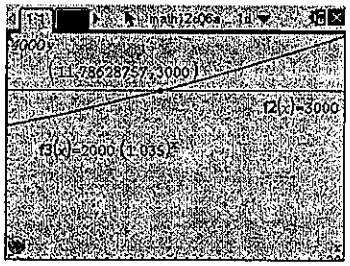
$y = \$2375.372...$

Investment A will have a value of \$1914.42 and investment B will have a value of \$2375.37.

d) It took 15 years for account A to have \$3000.



It took 12 years for account B to have \$3000.



2. a)

Investment	A	B	C
Initial amount	3000.00	3000.00	3000.00
12 months	3249.30	3185.04	3120.00
24 months	3514.98	3381.48	3244.80
36 months	3804.73	3590.05	3374.60
48 months	4118.36	3811.47	3509.58

A. $y = 3000\left(1+\frac{0.08}{4}\right)^x$

$y = 3000(1.02)^x$

B. $y = 3000\left(1+\frac{0.06}{12}\right)^x$

$y = 3000(1.005)^x$

C. $y = 3000\left(1+\frac{0.04}{1}\right)^{x}$

$y = 3000(1.04)^{x}$

b) e.g., Yes, the base is 1 + the annual interest rate divided by the number of compounding periods in a year.

c) A. $y = 3000(1.02)^{x}$

$y = 3000(1.02)^{5\times4}$

$y = 3000(1.02)^{20}$

$y = 3000(1.485...)$

$y = \$4457.842...$

The future value of investment A will be $4457.85.

B. $y = 3000(1.005)^{x}$

$y = 3000(1.005)^{12\times5}$

$y = 3000(1.005)^{60}$

$y = 3000(1.348...)$

$y = 4046.550...$

The future value of investment B will be $4046.55.

C. $y = 3000(1.04)^{x}$

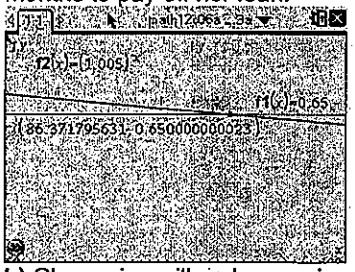
$y = 3000(1.04)^{5}$

$y = 3000(1.216...)$

$y = \$3649.958...$

The future value of investment C will be $3649.96.

3. a) It will take Charmaine 86.371... or 87 months to pay off her loan.



b) Charmaine will end up paying about $43 185.90. I determined my answer by multiplying the monthly payment by the number of months (86.37).

c) The principle amount that Charmaine initially borrowed must be known in order to determine the total amount of interest.

4. Use the equation $y = 150(1.04)^{x}$, substitute 1, 2, 3, 4, and 5 for x, and solve for y. By doing this, I get the following costs for the shoes:
After 1 year: $156
After 2 years: $162.24
After 3 years: $168.73
After 4 years: $175.48
After 5 years: $182.50

5. a)

End of Year	Amount Owed ($)	Increase in Amount Owed (%)
1	2157.84	
2	2330.47	8.000...
3	2516.90	7.999...
4	2718.26	8.000...

From the table above, the annual interest rate is about 8%, which is approximately the ratio between successful amounts owed.

b) To solve for the cost of the dirt bike, use the amount owed at the first year divided by 1.08.

$2157.84 = P(1.08)^{1}$

$P = \$1998$

The dirt bike cost $1998.

6. a) Using a graphing calculator, the function is $y = (7.628...)(1.742...)^{x}$.

b) i) $y = (7.628...)(1.742...)^{0.5}$

$y = (7.628...)(1.320)$

$y = \$10.069...$

The card had a value of about $10.07 6 months after purchase.

ii) $y = (7.628...)(1.742...)^{1.5}$

$y = (7.628...)(2.300...)$

$y = \$17.546...$

The card had a value of about $17.55 18 months after purchase.

iii) $y = (7.628...)(1.742...)^{2.5}$

$y = (7.628...)(4.008...)$

$y = \$30.575...$

The card had a value of about $30.58 30 months after purchase.

iv) $y = (7.628...)(1.742...)^{3.5}$

$y = (7.628...)(6.984...)$

$y = \$53.279...$

The card had a value of about $53.28 42 months after purchase.

7. a) e.g., First divide amounts in consecutive rows to determine the base, then use the initial amount and the base to write an exponential formula.

b)

Time (years)	Amount ($)	Increase in Amount (%)
0	1500.00	
1	1560.00	4
2	1622.40	4
3	1687.30	4.000...
4	1754.79	3.999...

The annual interest rate is 4%.

c) The function, which we must use, is
$y = 1500(1.04)^x$.
$y = 1500(1.04)^{10}$
$y = 1500(1.480...)$
$y = \$2220.366...$
Paula will have $2220.37 in her account after 10 years.

8. a) The equation that represents Carl's investment is $A(x) = 10\,000(1.05)^x$.
To determine the value of Carl's investment as the end of each of the next 4 years, substitute 1, 2, 3, and 4 for x into the above equation, and solve for y.

$y = 10000(1.05)^1$

$y = \$10500$

$y = 10000(1.05)^2$

$y = \$11025$

$y = 10000(1.05)^3$

$y = \$11576.25$

$y = 10000(1.05)^4$

$y = \$12155.062...$

Carl's investment will be worth $10 500 at the end of year 1, $11 025 at the end of year 2, $11 576.25 at the end of year 3, and $12 155.06 at the end of year 4.

b) $y = 10\,000(1.05)^x$

c) $y = 10000(1.05)^8$

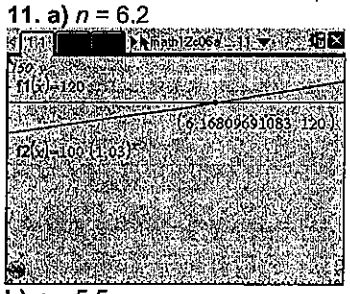
$y = \$14\,774.554...$

The value of Carl's investment after 8 years will be $14 774.55.

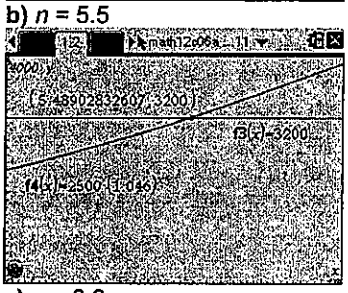
9. e.g., If interest is compounded k times per year, then replace i with $\frac{i}{k}$. For example, if the interest rate is 12% per year, compounded monthly (that is, 12 times per year), then the formula is $A(n) = P\left(1+\frac{0.12}{12}\right)^{12n}$ or $P(1.01)^{12n}$ where n is the number of years.

10. a) $A(n) = 1000(1.03)^n$, where n is the number of 6-month periods.
b) $A(n) = 1500(1.004...)^n$, where n is the number of months.
c) $A(n) = 4200(1.000\,865\,4...)^n$, where n is the number of weeks.
d) $A(n) = 2500(1.000\,068\,5...)^n$, where n is the number of days.

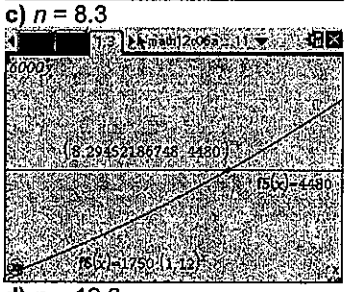
11. a) $n = 6.2$



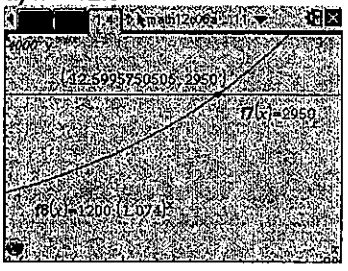
b) $n = 5.5$



c) $n = 8.3$



d) $n = 12.6$



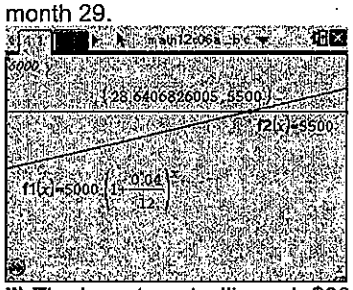
12. a) Value at the end of 2nd year:
35 000(0.8) = $28 000
Value at the end of 3rd year:
28 000(0.8) = $22 400
Value at the end of 4th year:
22 400(0.8) = $17 920
Value at the end of 5th year:
17 920(0.8) = $14 336
Value at the end of 6th year:
14 336(0.8) = $11 468.80

Value at the end of 7th year:
11 468.80(0.8) = $9175.04
b) Let x represent the number of years since the start of the 2nd year. Let y represent the value of the equipment. a is the value of the equipment at the start of the 2nd year $35 000.
b is the percent of the value of the equipment that is not depreciated each year 80% or 0.8.
The exponential regression function is
$y = 35\,000(0.8)^x$.
c) 10 – 1 = 9
9 years have passed since the start of the 2nd year.
$y = 35\,000(0.8)^9$
$y = 35\,000(0.134...)$
$y = \$4697.620...$
Merielle's equipment will be worth $4697.62 10 years after the purchase date.

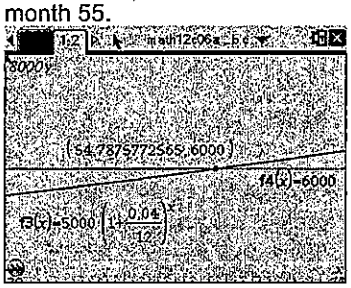
13. a)

Number of Months	Value of Investment
0	$5000
1	$5016.67
2	$5033.39
3	$5050.17
4	$5067.00
5	$5083.89

b) i) The investment will reach $5500 during month 29.



ii) The investment will reach $6000 during month 55.



c) i) $y = 5000\left(1+\frac{0.04}{12}\right)^x$

$y = 5000(1.003...)^x$

$y = 5000(1.003...)^{60}$

$y = 5000(1.220...)$

$y = \$6104.982...$

There will be $6104.99 in the account after 5 years.

ii) $y = 5000(1.003...)^{120}$

$y = 5000(1.490...)$

$y = \$7454.163...$

There will be $7454.17 in the account after 10 years.

14. Option A: $y = 1249.99(1+0.07)^2$

$y = 1249.99(1.07)^2$

$y = 1249.99(1.144...)$

$y = \$1431.113...$

Option B: $y = 1249.99\left(1+\frac{0.06}{12}\right)^{24}$

$y = 1249.99(1+0.005)^{24}$

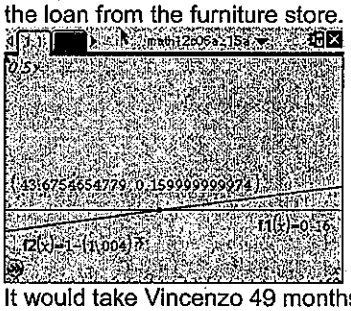
$y = 1249.99(1.005)^{24}$

$y = 1249.99(1.127...)$

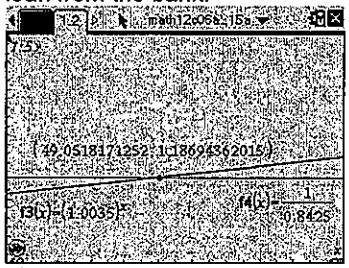
$y = \$1408.938...$

Denise should choose option A because her investment will be worth $1431.11, which is more than option B's $1408.94.

15. a) It would take Vincenzo 44 months to repay the loan from the furniture store.

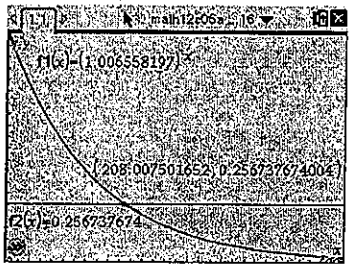


It would take Vincenzo 49 months to repay the loan from the bank.

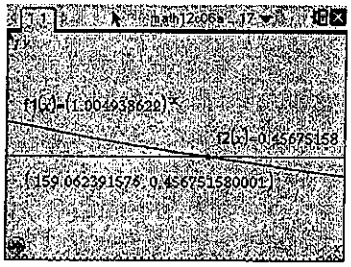


b) e.g., early payment options, effect on future credit rating,
affordable monthly payment

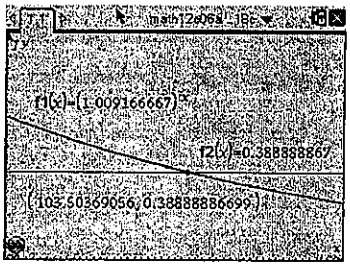
16. It will take Chao 208 months, or 17 years and 4 months, to repay the debt.



17. It will take Paul 160 months to pay off the debt.



18. a) Emma paid $24 000 for her new car.
b) $1 - 0.85 = 0.15$



$(0.15)(100\%) = 15\%$
Emma's grandfather applied an annual depreciation rate of 15% to her car.
c) It will take Emma 103.503... or 104 months to pay off the loan.
d) $A = 300(103.503...)$

$A = \$31051.107...$

$I = A - P$
$I = 31\,051.107... - 20\,000$
$I = \$11\,051.107...$
Emma will end up paying $11 051.11 in interest on the loan.

e) $A\left(\frac{104}{12}\right) = 24\,000(0.85)^{\frac{103.503...}{12}}$

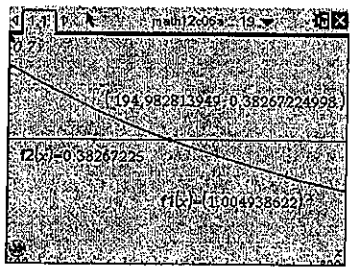
$A\left(\frac{104}{12}\right) = 24\,000(0.85)^{8.625...}$

$A\left(\frac{104}{12}\right) = 24\,000(0.246...)$

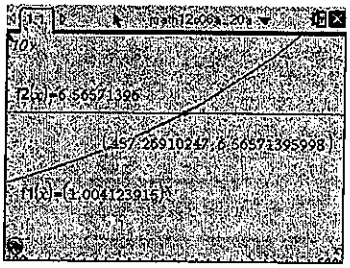
$A\left(\frac{104}{12}\right) = \$5907.827...$

Emma's car will be worth $5907.83.

19. It will take 16 years and 3 months to repay the mortgage.



20. a) It will take 38 years and 2 months for them to pay off the mortgage.



b) Jason and Jamie will have paid $411 542 of which $226 542.19 will be interest.

21. a) $A(60) = 100\,000\left(1 + \frac{0.045}{12}\right)^{60}$

$A(60) = 100\,000(1 + 0.003...)^{60}$

$A(60) = 100\,000(1.003...)^{60}$

$A(60) = 100\,000(1.251...)$

$A(60) = \$125179.582...$

Clorinda will have $125 179.58 in her account after 5 years.
b) It will take Clorinda 122 months to pay off the rest of her mortgage.

22. a) e.g. $P = \frac{5000}{\left(1 + \frac{0.12}{12}\right)^{12}}$

b) e.g., You could calculate the value of an investment in later years once you have calculated the principal in part a). For example, you could determine the principal for an investment that is worth $10 000 after 10 years where interest is 9% compounded monthly. The solution is $4079.37.

23. a). Disagree. e.g., Interest factors of the form $(1 + i)$ are multiplied, not added, so it's unclear which option will be best.

b) Option A:
First year: 1.06(40 000) = $42 400
Second year: 1.04(42 400) = $44 096
Third year: 1.02(44 096) = $44 977.92
Total Paid:
42 400 + 44 096 + 44 977.92 = $131 473.92
Option B:
First year: 1.04(40 000) = $41 600
Second year: 1.04(41 600) = $43 264
Third year: 1.04(43 264) = $44 994.56
Total Paid:
41 600 + 43 264 + 44 994.56 = $129 858.56
Option C:
First year: 1.02(40 000) = $40 800
Second year: 1.08(40 800) = $44 064
Third year: 1.02(44 064) = $44 945.28
Total Paid:
40 800 + 44 064 + 44 945.28 = $129 809.28

I should choose option A. It has the highest total paid over the three years. Although option B will have a higher permanent salary after the three years, it would take approximately 97 years for the higher salary of option B to make up for the higher amount paid by option A over the three years:
44 994.56 – 44 977.92 = $16.64
131 473.92 – 129 858.56 = $1615.36
1615.36 ÷ 16.64 = 97 years

Math in Action, page 399

I divided 72 by the annual interest rate.

$$\frac{72}{3} = 24$$

It will take about 24 years for the $2000 investment to double.
Verify:
$A = P(1 + i)^n$
$A = 2000(1 + 0.03)^{24}$
$A = 4065.59$
This is close to double $2000.

At 48 years, the future value will be about (2)($4000) = $8000.

Verify:
$A = P(1 + i)^n$
$A = 2000(1 + 0.03)^{48}$
$A = 8264.50$
This is close to double $4000.

- Both of my estimates are fairly accurate. However, my first estimate is closer to the actual value than my second estimate.
- Since the compounding periods are semi-annually, quarterly, and monthly, the doubling times are in terms of half years, quarter years, and months.

I started by determining the interest rate per compounding period.
3%/a compounded semi-annually is 1.5%/half year.
Then I applied the Rule of 72.

$$\frac{72}{1.5} = 48$$

It will take about 48 half years for the $2000 investment to double.
Verify:
$A = P(1 + i)^n$
$A = 2000(1 + 0.015)^{48}$
$A = 4086.96$
This is close to double $2000.

3%/a compounded quarterly is 0.75%/quarter.
Applying the Rule of 72,

$$\frac{72}{0.75} = 96$$

It will take about 96 quarters for the $2000 investment to double.
Verify:
$A = P(1+i)^n$
$A = 2000(1+ 0.0075)^{96}$
$A = 4097.84$
This is close to double $2000.

3%/a compounded monthly is 0.35%/month
Applying the Rule of 72,

$$\frac{72}{0.25} = 288$$

It will take about 288 months for the $2000 investment to double.
Verify:
$A = P(1 + i)^n$
$A = 2000(1 + 0.0025)^{288}$
$A = 4105.18$
This is close to double $2000.

- When the compounding period is annual, the estimate is the most accurate. The less the compounding period, the less accurate the estimate is.

Chapter Self-Test, page 400

1. To match the functions with the graphs, look at the *y*-intercepts of the functions and of the graphs. This is because each function and each graph has a unique *y*-intercept.
a) It is a decreasing exponential function with *y*-intercept of 0.2 so it must match with ii.
b) It is a decreasing exponential function with *y*-intercept of 3, so it must match with iv.
c) It is a increasing exponential function with *y*-intercept of 2, so it must match with iii.
d) It is a increasing exponential function with *y*-intercept of 0.3, so it must match with i.

2. a) $5^{(x-2)} = \left(\frac{1}{125}\right)^x$

$5^{(x-2)} = \left[(5^3)^{-1}\right]^x$

$5^{(x-2)} = 5^{-3x}$

$x - 2 = -3x$

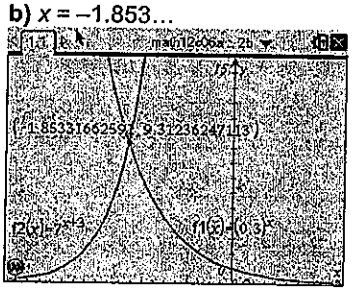
$4x = 2$

$x = \frac{1}{2}$

LS	RS
$5^{\left(\frac{1}{2}-2\right)}$	$\left(\frac{1}{125}\right)^{\frac{1}{2}}$
$5^{\left(-\frac{3}{2}\right)}$	$\sqrt{\frac{1}{125}}$
$\frac{1}{5^{\frac{3}{2}}}$	$\frac{\sqrt{1}}{\sqrt{125}}$
$\frac{1}{\sqrt{5^3}}$	$\frac{1}{\sqrt{125}}$
$\frac{1}{\sqrt{125}}$	

LS = RS

b) $x = -1.853...$



LS	RS
$(0.3)^{-1.853...}$	$7^{(-1.853...+3)}$
$\frac{1}{(0.3)^{1.853...}}$	$7^{1.146...}$
$\frac{1}{0.107...}$	9.312...
9.312...	

LS = RS

3. a)

30-min Time Intervals	Number of Yeast Cells
0	300
1	600
2	1200
3	2400
4	4800

b) $y = 300(2)^{\frac{t}{30}}$, where t is the number of minutes after 1:00 p.m., and y is the number of yeast cells.

c) i) 4 h = 240 min.

$y = 300(2)^{\frac{240}{30}}$

$y = 300(2)^8$

$y = 300(256)$

$y = 76800$

There will be 76 800 yeast cells after 4 h.

ii) 12 h = 720

$y = 300(2)^{\frac{720}{30}}$

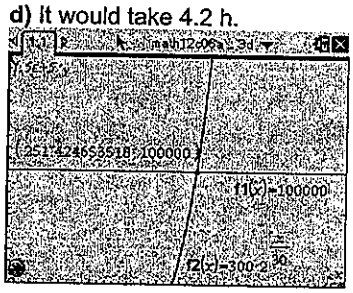
$y = 300(2)^{24}$

$y = 300(16777216)$

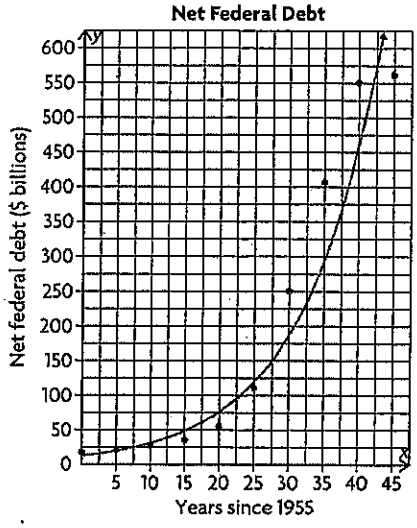
$y = 5033164800$

There will be 5 033 164 800 yeast cells after 12 h.

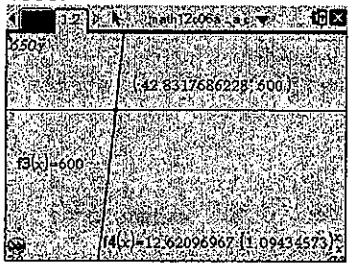
d) It would take 4.2 h.



4. a) The exponential regression function is $y = 12.620...(1.094...)^x$, where y is the debt in billions of dollars and x is the number of years after 1955.



b) $1988 - 1955 = 33$
$y = 12.620...(1.094...)^{33}$
$y = 12.620...(19.592...)$
$y = \$247.282...$ billion
The net federal debt was $247.28 billion.
c) Assuming the same growth rate and that these figures were taken at the end of the year, the net federal debt should have reached $600 billion in 1998.



5. a) It will take Angie 56.640... or 57 months to repay the loan.
b) $A = 400(56.640...)$
$A = \$22\,656.366...$
$I = A - P$
$I = 22\,656.365... - 21\,400$
$I = \$1256.365...$
The total amount of interest that Angie will pay on the loan is $1256.37.

Chapter Review, page 402

1. a) e.g., The end behaviours of exponential functions extend from QII to QI. Polynomials have varying end behaviours.
b) e.g., The domain is always $\{x \mid x \in R\}$, the range is always $\{y \mid y > 0, y \in R\}$, and they all extend from QII to QI. a is the y-intercept.

2. a) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
y-Intercept: $y = 9$; end behaviour: QII to QI
This is a decreasing function.
b) To make the function into an increasing exponential function, the b should be changed so its value is greater than 1.

3. a) number of x-intercepts: 0; y-intercept: 125
end behaviour: QII to QI; domain: $\{x \mid x \in R\}$
range: $\{y \mid y > 0, y \in R\}$
The function decreases.
b) number of x-intercepts: 0; y-intercept: 0.12
end behaviour: QII to QI; domain: $\{x \mid x \in R\}$
range: $\{y \mid y > 0, y \in R\}$.
The function decreases.
c) number of x-intercepts: 0; y-intercept: 1
end behaviour: QII to QI; domain: $\{x \mid x \in R\}$
range: $\{y \mid y > 0, y \in R\}$
The function increases.
d) number of x-intercepts: 0; y-intercept: 0.85
end behaviour: QII to QI; domain: $\{x \mid x \in R\}$
range: $\{y \mid y > 0, y \in R\}$
The function increases.

4. number of x-intercepts: 0; y-intercept: 1
end behaviour: QII to QI; domain: $\{x \mid x \in N\}$
range: $\{y \mid y > 0, y \in N\}$
The function increases.

5. $4(3)^{\frac{3x}{5}} = 108$

$$4(3)^{\frac{3x}{5}} = 4(27)$$
$$(3)^{\frac{3x}{5}} = (3)^3$$
$$\frac{3x}{5} = 3$$
$$3x = 15$$
$$x = 5$$

6. a) $3^{2x+7} = 27^{x+2}$

$$3^{2x+7} = \left(3^3\right)^{x+2}$$
$$3^{2x+7} = 3^{3x+6}$$
$$2x + 7 = 3x + 6$$
$$-x = -1$$
$$x = 1$$

LS	RS
$3^{2(1)+7}$	27^{1+2}
3^9	27^3
19 683	19 683

LS = RS

b) $\left(\frac{1}{8}\right)^{3x} = 16^{3-x}$

$\left(8^{-1}\right)^{3x} = 16^{3-x}$

$\left[\left(2^3\right)^{-1}\right]^{3x} = \left(2^4\right)^{3-x}$

$2^{-9x} = 2^{12-4x}$

$-9x = 12 - 4x$

$-5x = 12$

$x = -\frac{12}{5}$

LS	RS
$\left(\frac{1}{8}\right)^{3\left(-\frac{12}{5}\right)}$	$16^{3-\left(-\frac{12}{5}\right)}$
$\left(\frac{1}{8}\right)^{-\frac{36}{5}}$	$16^{\frac{27}{5}}$
$\frac{1}{\left(\frac{1}{8}\right)^{\frac{36}{5}}}$	$\sqrt[5]{16^{27}}$
3 178 688.029	3 178 688.029

LS = RS

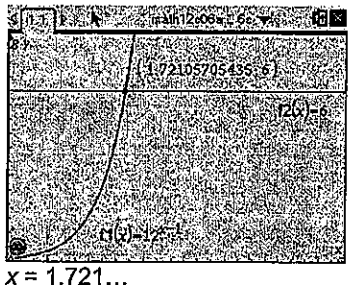
c) $2^{x-1} = \left(\frac{1}{6}\right)^{x-2}$

$2^{x-1} = \left(\frac{1}{6}\right)^{x-1-1}$

$2^{x-1} = \left(\frac{1}{6}\right)^{x-1} \cdot \left(\frac{1}{6}\right)^{-1}$

$\frac{2^{x-1}}{\left(\frac{1}{6}\right)^{x-1}} = \left(\frac{1}{6}\right)^{-1}$

$12^{x-1} = 6$

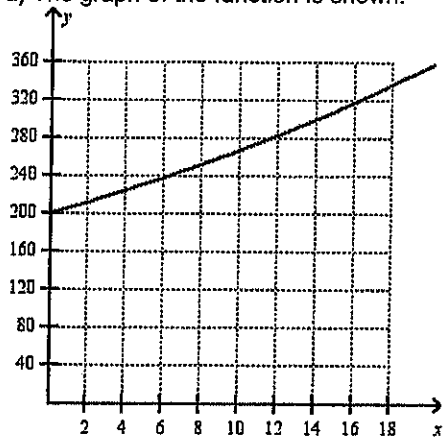


$x = 1.721...$

LS	RS
$2^{1.721...-1}$	$\left(\frac{1}{6}\right)^{1.721...-2}$
$2^{0.721...}$	$\left(\frac{1}{6}\right)^{-0.278...}$
1.648...	$\frac{1}{\left(\frac{1}{6}\right)^{0.278}}$
	1.648...

LS = RS

7. a) The graph of the function is shown:



i) e.g., One possible prediction is that in 12 min, there will be 283 bacteria in the sample.

ii) $y = 200(2)^{\frac{720}{24}}$

$y = 2.147... \times 10^{11}$

There will be about 2.15×10^{11} bacteria in the sample in 12 h.

b) $204\,800 = 200(2)^{\frac{x}{24}}$

$1024 = 2^{\frac{x}{24}}$

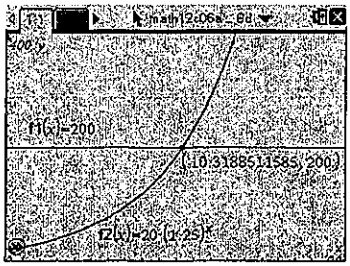
$2^{10} = 2^{\frac{x}{24}}$

$10 = \frac{x}{24}$

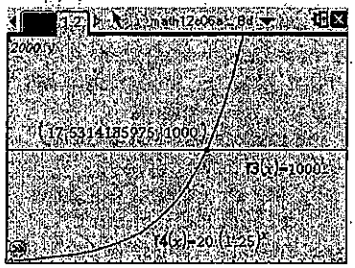
$x = 240 \text{ min}$

It takes 240 min for the sample to reach 204 800 *E. coli.*

8. a) 20 yeast cells were in the initial sample.
b) 40 cells were in the sample after 3 hours.
c) $y = 20(1.26)^x$
$y = 127.055...$
There will be 127 cells in the sample after 8 hours.
d) i) The sample will reach 200 cells during hour 10.



ii) The sample will reach 1000 cells during hour 17.



9. a) i) $f(5) = 10.00(0.944)^5$

$f(5) = 10.00(0.749...)$

$f(5) = 7.496...\text{kg}$

7.50 kg of DDT would be present 5 years after the spraying.

ii) $f(15) = 10.00(0.944)^{15}$

$f(15) = 10.00(0.421...)$

$f(15) = 4.212...\text{kg}$

4.21 kg of DDT would be present 15 years after the spraying.

b) The mass of DDT would reach 5 kg about 12 years after the spraying.

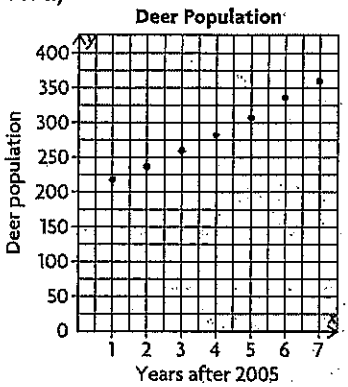
c) It would take 40 years for less than 1 kg of DDT to be present.

10. a) Let x be the number of months since April 1, 1896.
Let y be the population of Dawson City.
$a = 1000$
$b = 3$
Tripling time: 3 months
An exponential equation that models the population growth is $y = 1000(3)^x$, where y is the population and x is the number of quarters after April 1, 1896.

b) Domain: $\{x \mid 0 \le x \le 3, x \in \mathbb{N}\}$ (quarters)
Range: $\{y \mid 1000 \le y \le 27\,000, y \in \mathbb{N}\}$ (population)

c) The population of Dawson City in mid-May of 1896 was 1732, and the population of Dawson City in mid-August of 1896 was 5196. To determine the answer for mid-May, I substituted in 1.5 for x for my equation. To determine the answer for mid-August, I substituted in 4.5 for x for my equation. These two points are midpoints of the first two quarters, and so correspond to $x = 0.5$ and $x = 1.5$.

11. a)



b) Using the graphing calculator, the exponential regression function is $y = 200.798...(1.088...)^x$.

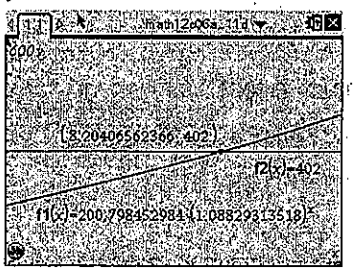
c) $y = 200.798...(1.088...)^{10}$

$y = 200.798...(2.330...)$

$y = 467.971...$

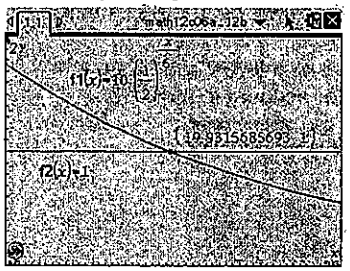
The deer population 10 years after 2005 will be 468.

d) 200.798...(2) = 402 (rounded).
Assuming the deer population numbers were recorded at the beginning of the year, I would expect the deer population to have doubled in the year 2014.



12. a) Let x represent the number of hours that have passed. Let y represent the amount in mg of technetium-99 that remains. The half-life equation is $y = 10\left(\frac{1}{2}\right)^{\frac{x}{6.0}}$.

b) The initial 10 mg will have decayed to less than 1 mg during the 20th hour.



13. a) $A(8000) = 100\left(\frac{1}{2}\right)^{\frac{8000}{5730}}$

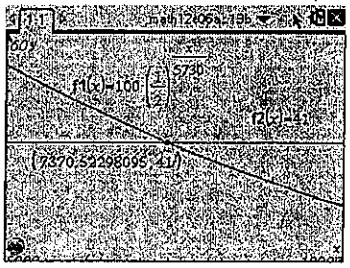
$A(8000) = 100\left(\frac{1}{2}\right)^{1.396...}$

$A(8000) = 100(0.379...)$

$A(8000) = 39.993...\%$

38% of the initial carbon-14 would be present in the tools.

b) The age of the tools were 7400 years old.



14. a) $A(n) = 8000\left(1+\frac{0.06}{12}\right)^{n}$

$A(n) = 8000(1.005)^{n}$

b) i) $A(30) = 8000\left(1+\frac{0.06}{12}\right)^{30}$

$A(30) = 8000(1+0.005)^{30}$

$A(30) = 8000(1.005)^{30}$

$A(30) = 8000(1.161...)$

$A(30) = \$9291.200...$

After 30 months, there will be \$9291.20 in Kaja's account.

ii) $A(48) = 8000\left(1+\frac{0.06}{12}\right)^{48}$

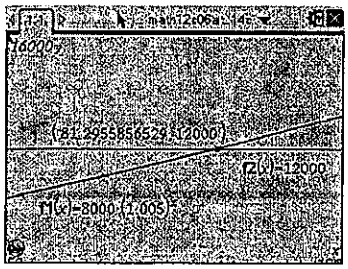
$A(48) = 8000(1.005)^{48}$

$A(48) = 8000(1.270...)$

$A(48) = \$10163.913...$

After 48 months, there will be 10 163.91 in Kaja's account.

c) Assuming the same interest rate, Kaja will have \$12 000 in her account in month 82.



15. $A(n) = 1000(1+0.05)^{n}$

$A(18) = 1000(1+0.05)^{18}$

$A(18) = 1000(1.05)^{18}$

$A(18) = 1000(2.406...)$

$A(18) = \$2406.619...$

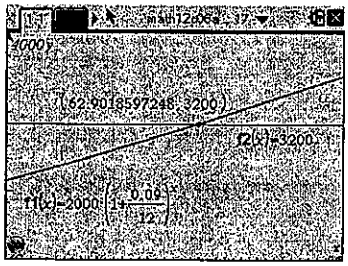
Zachary had \$2406.62 in this account on his 18th birthday.

16. Changing the compounding period changes how many times interest is compounded over a period of time. The more times interest is compounded, the more often the investment will grow. For example, if you have interest compounded daily, the investment will grow 365 times a year, but if you have interest compounded annually, the investment will grow once a year. Thus, increasing the number of compounding periods increases the final amount of the investment.

17. a) The equation we need to use is

$A(n) = 2000\left(1+\frac{0.09}{12}\right)^{n}$.

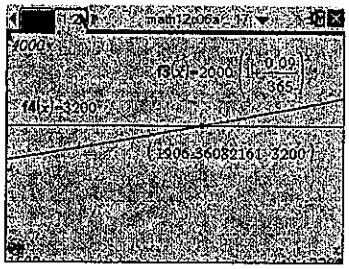
The investment will grow to \$3200 after 63 months.



b) The equation we nee to use is

$A(n) = 2000\left(1+\frac{0.09}{365}\right)^{n}$

The investment will grow to \$3200 after 1907 days.



18. a) The exponential regression equation is $y = 32\,094.600...(0.849...)^{x}$.

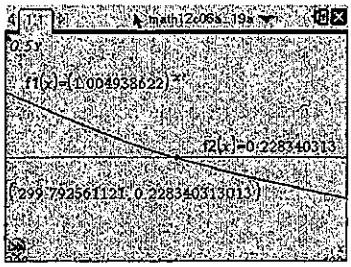
b) The initial selling price of the truck was \$32 094.60.

c) $1 - 0.849... = 0.150...$

$0.150... \cdot 100\% = 15.098...\%$

The truck depreciates in value by 15% each year.

19. a) Use a graphing calculator to determine the value of n. $n = 299.792...$



This is the number of months, but we want the number of years.

$$\frac{299.792...}{12} = 24.982...$$

It will take Daryl 25 years to pay off his mortgage.

b) $A = 1600(299.792...)$

$A = \$479\,668.097...$

$I = A - P$

$I = 479\,668.097... - 250\,000$

$I = \$229\,668.097...$

The total interest that Darryl will have paid by the time he has paid off his mortgage is $229 668.10.

Chapter Task, page 405

A. I found that the amount of caffeine in one espresso is about 100 mg. So, a triple espresso will have 300 mg of caffeine.

B. I chose to use a half-life of 6 h for caffeine in the human body. I substituted $h = 6$ into the general half-life equation.

$$A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{6}}$$

where t represents time, in hours

C. To determine the exponential function that models the amount of caffeine in Maria's body after she drinks one triple espresso, I substituted $A_0 = 300$ into my half-life equation from prompt B.

$A(t) = 300\left(\frac{1}{2}\right)^{\frac{t}{6}}$, where t represents time, in hours.

D. To determine the amount of caffeine in Maria's body 24 h after her first triple espresso, I substituted $t = 24$ into my exponential equation from prompt C.

$$A(t) = 300\left(\frac{1}{2}\right)^{\frac{t}{6}}$$

$$A(t) = 300\left(\frac{1}{2}\right)^{\frac{24}{6}}$$

$$A(t) = 300\left(\frac{1}{2}\right)^{4}$$

$A(t) = 18.75mg$

Maria will have 18.75 mg of caffeine in her system 24 h after drinking her first triple espresso.

E. I chose to use a spreadsheet to calculate the amount of caffeine in Maria's system during the week.

	A	B	C
1	Day of the Week	Mon.	Tue.
2	Starting amount of caffeine (mg)	300.0000	318.7500
3	Amount of caffeine after 24 h	18.7500	19.9219

4	Day of the Week	Wed.	Thu.
5	Starting amount of caffeine (mg)	319.9219	319.9951
6	Amount of caffeine after 24 h	19.9951	19.9997

8	Day of the Week	Fri.	Sat.
9	Starting amount of caffeine (mg)	319.9997	20.0000
10	Amount of caffeine after 24 h	20.0000	1.2500

On Saturday morning, Maria will have about 20.00 mg of caffeine in her system. If she does not drink coffee (or any other caffeinated drink) on Saturday, she will have only 1.25 mg of caffeine in her system by Saturday evening.

Chapter 6 Diagnostic Test, TR page 371

1. a) $4^3 = 64$

b) $\left(\frac{1}{3}\right)^4 = \frac{1^4}{3^4}$

$\left(\frac{1}{3}\right)^4 = \frac{1}{81}$

c) $(121)^{\frac{1}{2}} = \sqrt{121}$

$(121)^{\frac{1}{2}} = 11$

d) $(5)^{-4} = \frac{1}{5^4}$

$(5)^{-4} = \frac{1}{625}$

$(5)^{-4} = 0.0016$

2. a) $3 \cdot 7 \cdot 7 = 3(7)^2$

b) $\frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^2$

c) $a \cdot a \cdot a \cdot b = a^3b$

d) $\frac{3}{x} \cdot \frac{3}{x} \cdot \frac{3}{x} \cdot y \cdot y''? \frac{y^4}{x^5}$

3. a) $27 = 3^3$

b) $64 = 8^2$

$64 = 4^3$

$64 = 2^6$

c) $625 = 5^4$

d) $169 = 13^2$

4. a) $6^2 \cdot 6^{-4} \cdot 6^5 = \frac{6^7}{6^4}$

$6^2 \cdot 6^{-4} \cdot 6^5 = 6^3$

b) $\frac{1}{3} \cdot \left(\frac{1}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^{-1} = \left(\frac{1}{3}\right)^3$

c) $2^5 \cdot 2^{\frac{1}{2}} \cdot 2^{-2} = \frac{2^5 \cdot 2^{\frac{1}{2}}}{2^2}$

$2^5 \cdot 2^{\frac{1}{2}} \cdot 2^{-2} = 2^3 \cdot 2^{\frac{1}{2}}$

$2^5 \cdot 2^{\frac{1}{2}} \cdot 2^{-2} = 2^{\frac{7}{2}}$

d) $\left(\frac{1}{5}\right)^4 \cdot \left(\frac{1}{5}\right)^{-2} \cdot 5^3 \cdot 5 = \frac{5^2 \cdot 5^3 \cdot 5}{5^4}$

$\left(\frac{1}{5}\right)^4 \cdot \left(\frac{1}{5}\right)^{-2} \cdot 5^3 \cdot 5 = \frac{5^6}{5^4}$

$\left(\frac{1}{5}\right)^4 \cdot \left(\frac{1}{5}\right)^{-2} \cdot 5^3 \cdot 5 = 5^2$

5. a) $\frac{7^2}{7^5} = 7^{-3}$

b) $\frac{4^{-2}}{4^{-7}} = 4^5$

c) $\frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^{-5}} = \left(\frac{1}{2}\right)^7$

d) $\frac{6^{-2}}{6} = 6^{-3}$

6. a) $\frac{2^{-2} \cdot 2^6}{2^4} = 2^{-2} \cdot 2^2$

$\frac{2^{-2} \cdot 2^6}{2^4} = 1$

b) $\frac{5 \cdot 5^2}{\left(\frac{1}{5}\right)^2} = 5 \cdot 5^2 \cdot 5^2$

$\frac{5 \cdot 5^2}{\left(\frac{1}{5}\right)^2} = 5^5$

$\frac{5 \cdot 5^2}{\left(\frac{1}{5}\right)^2} = 3125$

c) $\frac{4^6 \cdot 3^2}{4^4 \cdot 3^{-1}} = 4^2 \cdot 3^3$

$\frac{4^6 \cdot 3^2}{4^4 \cdot 3^{-1}} = 432$

d) $\frac{6^5 \cdot 2^3}{6^3 \cdot 6^{-2}} = \frac{6^5 \cdot 2^3}{6}$

$\frac{6^5 \cdot 2^3}{6^3 \cdot 6^{-2}} = 6^4 \cdot 2^3$

$\frac{6^5 \cdot 2^3}{6^3 \cdot 6^{-2}} = 10\,368$

7. a) $6 - 3x = 8$

$-3x = 8 - 6$

$-3x = 2$

$x = -\frac{2}{3}$

LS	RS
$6 - 3x$	8
$6 - 3\left(-\frac{2}{3}\right)$	
$6 + 2$	
8	

LS = RS

b) $2x-\frac{1}{2}=5$

$2x=5+\frac{1}{2}$

$2x=\frac{11}{2}$

$x=\frac{11}{4}$

LS	RS
$2x-\frac{1}{2}$	5
$2\left(\frac{11}{4}\right)-\frac{1}{2}$	
$\frac{11}{2}-\frac{1}{2}$	
5	

LS = RS

c) $5x+\frac{1}{3}=8x$

$5x-8x=-\frac{1}{3}$

$-3x=-\frac{1}{3}$

$x=\frac{1}{9}$

LS	RS
$5x+\frac{1}{3}$	8x
$5\left(\frac{1}{9}\right)+\frac{1}{3}$	$8\left(\frac{1}{9}\right)$
$\frac{5}{9}+\frac{3}{9}$	$\frac{8}{9}$
$\frac{8}{9}$	

LS = RS

d) $13-\left(\frac{1}{4}\right)x=3x$

$-\left(\frac{1}{4}\right)x-3x=-13$

$-\left(\frac{13}{4}\right)x=-13$

$x=4$

LS	RS
$13-\left(\frac{1}{4}\right)x$	3x
$13-4\left(\frac{1}{4}\right)$	3(4)
13 – 1	12
12	

LS = RS

8. a) No, a constant multiple was not used to generate the numbers in this set. This can be seen in the ratio between each successive number.

Number	Ratio
1	
2	2
4	2
7	1.75
11	1.571...

b) Yes, a constant multiple of 2 was used to generate the numbers in this set. This can be seen in the ratio between each successive number.

Number	Ratio
3	
6	2
12	2
24	2
48	2

c) Yes, a constant multiple of $\frac{1}{3}$ was used to generate the numbers in this set. This can be seen in the ratio between each successive number.

Number	Ratio
36	
12	$\frac{1}{3}$
4	$\frac{1}{3}$
$\frac{4}{3}$	$\frac{1}{3}$
$\frac{4}{9}$	$\frac{1}{3}$

d) No, a constant multiple was not used to generate the numbers in this set. This can be seen in the ratio between each successive number.

Number	Ratio
4	
9	2.25
16	1.777...
25	1.562...
36	1.44

Review of Terms and Connections, TR page 373

1. a) It would match with iv) because the two coordinates are the *y*-intercept and *x*-intercept of $y = 4x - 5$.

y-intercept:

$y = 4(0) - 5$

$y = -5$

x-intercept:

$y = 4x - 5$

$0 = 4x - 5$

$5 = 4x$

$\frac{5}{4} = x$

b) This would match with vi) because the number of books collected in a year is a discrete number. The number of books cannot be continuous; it has to be a discrete unit in which each unit is separate. For example, it is impossible to have 0.5 books. It is possible to have 2 books. If this were to be graphed, it would look like a staircase because the line is not a continuous whole.

c) This matches with vii) because the domain of $y = x^3$ is all real numbers.

d) It would match with v) because the range of $y = x^2$ is all values that are equal and greater than zero.

e) This would match with iii) because for $y = x^2$, the function increases when x is greater than zero.

f) It would match with i) because the height of a person over time is continuous not discrete. It is a gradual increase. If the height of a person were graphed, it would be a continuous line without a break.

g) This matches with ii) because the function is linear since x has an exponent of 1.

2. a) The domain of this function is $\{t \mid t \geq 0, t \in \mathrm{R}\}$. This is because t represents time and the ball cannot have negative seconds after it falls off a balcony.

b) The domain of this function is $\{b \mid b \geq 0, b \in \mathrm{R}\}$. This is because b represents batches of cookies and it is impossible to bake negative batches of cookies. It is possible to bake zero or more batch of cookies.

3. a) The range of this function is $\{h(t) \mid 0 \leq h(t) \leq 3, h(t) \in \mathrm{R}\}$ because if the ball is falling off the balcony 3 m up, that must be its maximum height. The minimum height would be 0 m, where the ball hits the ground.

b) The range of this function is $\{n(b) \mid 0 \leq n(b), n(b) \in \mathrm{W}\}$ because it is impossible to have a negative number of cups of flour. The number of cups of flour must be zero and greater to be able to bake b batches of cookies.

4. a) $y = 3x + 6$

y-intercept:

$y = 3x + 6$

$y = 3(0) + 6$

$y = 6$

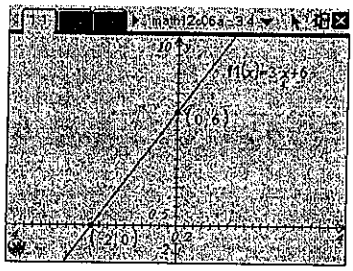
x-intercept:

$y = 3x + 6$

$(0) = 3x + 6$

$-3x = 6$

$x = -2$



The x-intercept is -2 and the y-intercept is 6.

b) $y = x^2 - 4$

y-intercept:

$y = x^2 - 4$

$y = (0)^2 - 4$

$y = -4$

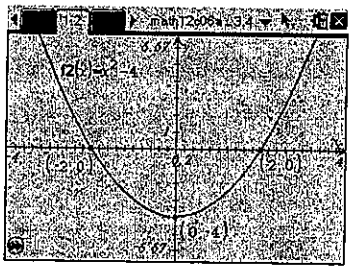
x-intercepts:

$y = x^2 - 4$

$(0) = x^2 - 4$

$4 = x^2$

$-2, 2 = x$



The x-intercepts are -2 and 2 while the y-intercept is 4.

c) $y = 2x^3$

y-intercept:

$y = 2x^3$

$y = 2(0)^3$

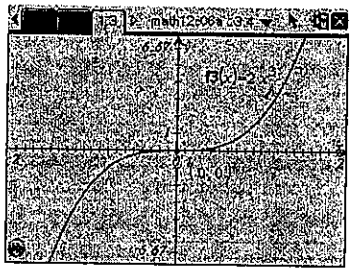
$y = 0$

x-intercept:

$y = 2x^3$

$(0) = 2x^3$

$0 = x$



The x-intercept is 0 and the y-intercept is 0.

5. a) The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \in R\}$ because it is a linear function.
b) The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \leq 8, y \in R\}$ because it is a quadratic function.
c) The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \in R\}$ because it is a cubic function.
d) The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \in R\}$ because it is a cubic function.

6. a) The function is increasing because the slope is positive.
b) The function is decreasing when $x > 0$ and increasing when $x < 0$.
c) The function is decreasing because the slope is negative.
d) The function is increasing because the slope is positive.

7. a) $y = -2x^2 + 8$

y-intercept:	x-intercept:
$y = -2x^2 + 8$	$y = -2x^2 + 8$
$y = -2(0)^2 + 8$	$(0) = -2x^2 + 8$
$y = 8$	$-8 = -2x^2$
	$4 = x^2$
	$\pm 2 = x$

The y-intercept is 8 and the x-intercepts are -2 and 2.

b) $y = x^3$

y-intercept:	
$y = x^3$	$y = x^3$
$y = (0)^3$	$(0) = x^3$
$y = 0$	$0 = x$

x-intercept:
The y-intercept is 0 and the x-intercept is 0.

c)

y-intercept:	x-intercept:
$y = \frac{1}{4}x - 2$	$(0) = \frac{1}{4}x - 2$
$y = \frac{1}{4}(0) - 2$	$2 = \frac{1}{4}x$
$y = 2$	$8 = x$

The y-intercept is 2 and the x-intercept is 8.

d)

y-intercept:	x-intercept:
$y = -4x + 3$	$0 = -4x + 3$
$y = -4(0) + 3$	$-3 = -4x$
$y = 3$	$\frac{3}{4} = x$

The y-intercept is 3 and the x-intercept is $\frac{5}{6}$.

8. Answers will vary, e.g., $y = -2x + 2$
For a linear function, the y-intercept will give a value for b in equations such as $y = mx + b$. From this, the coordinates of the x-intercept can be substituted in to calculate m.

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1.

Characteristics	$f(x) = 2(5)^x$	$f(x) = 3(2)^x$
Number of x-intercepts	No x-intercepts	No x-intercepts
y-intercept	2	3
End Behaviour	QII to QI	QII to QI
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y \geq 0, y \in R\}$	$\{y \mid y \geq 0, y \in R\}$
Increasing or Decreasing?	Increasing	Increasing

Characteristics	$f(x) = \left(\frac{1}{2}\right)^x$	$f(x) = 4\left(\frac{1}{3}\right)^x$
Number of x-intercepts	No x-intercepts	No x-intercepts
y-intercept	1	4
End Behaviour	QII to QI	QII to QI
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y \geq 0, y \in R\}$	$\{y \mid y \geq 0, y \in R\}$
Increasing or Decreasing?	Decreasing	Decreasing

2. a) i) The y-intercept is 5.
ii) The graph extends from quadrant II to quadrant I.
iii) The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq 0, y \in R\}$.
iv) The function is decreasing.
b) Since this function is decreasing, I know the parameter b is between 0 and 1.

3. I wrote the values of a and b. $f(x) = 2(10)^x$
$a = 2$ and $b = 10$
Since $a > 0$ and $b > 1$, I knew the exponential function is increasing and it domain is $\{x \mid x \in R\}$ and its range is $\{y \mid y \geq 0, y \in R\}$.
The number of x-intercepts is zero. This is because it is an exponential function that never crosses the x-axis.
The y-intercept is 2.
$y = 2(10)^x$
$y = 2(10)^{(0)}$
$y = 2(1)$
$y = 2$
There are no x-intercepts.
The end behaviour is QII to QI because the graph extends from quadrant II to quadrant I.

4. a) This function matches with ii) because it has a y-intercept of (0, 4) and it is increasing.
b) This function matches with iii) because it is an increasing function with a y-intercept of (0, 2).
c) This matches with i) because it is a decreasing function with a y-intercept of (0, 4).
d) This matches with iv) because it is a decreasing function with a y-intercept of 3.

5. a) This data represents an exponential function because it has a constant ratio of 2 for each pair of successful y-values.

y	Ratio
1	
2	2
4	2
8	2
16	2
32	2

b) This data does not represent an exponential function because it does not have a constant ratio for each pair of successful y-values.

y	Ratio
5	
2	0.4
1	0.5
2	2
5	2.5
10	2

c) This data does represent an exponential function because it has a constant ratio of 3 for each pair of successful y-values.

y	Ratio
$\frac{2}{3}$	
2	3
6	3
18	3
54	3
162	3

6. a) $y = 2^x$ **c)** $y = 2(3)^x$

7. a) $2(3)^x = 54$
$(3)^x = 27$
$(3)^x = 3^3$
$x = 3$

b) $2^{(x+1)} = \sqrt{32}$
$2^{(x+1)} = 32^{\frac{1}{2}}$
$2^{(x+1)} = (2^5)^{\frac{1}{2}}$
$2^{(x+1)} = 2^{\frac{5}{2}}$
$x+1 = \frac{5}{2}$
$x = \frac{3}{2}$

c) $25^{(x-3)} = \frac{1}{125}$
$5^{2(x-3)} = \frac{1}{5^3}$
$5^{2x-6} = 5^{-3}$
$2x-6 = -3$
$2x = 3$
$x = \frac{3}{2}$

8. a) Using a graphing calculator, the equation of the exponential regression function that models the data is $y = 2.305...(1.181...)^x$. The regression equation appears to fit the data very well. The algebraic model would be $y = 2.305...(1.181...)^x$, where y represents the number of dandelions and x represents the date in May.
b) Assuming that the population growth continues at the same rate, 401 dandelions will be growing in the yard on May 31.
$y = 2.305...(1.181...)^x$
$y = 2.305...(1.181...)^{31}$
$y = 401.425...$

9. a) $A = P(1+i)^n$

$$A = 4000\left(1+\frac{0.05}{4}\right)^4$$

$A = 4000(1.0125)^4$
$A = 4203.781...$
Joe will have $4203.78 in the account after 1 year.
b) It will take 4.5 years.